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Eliminate the Nonlinear Oscillations of the Modified duffing Equation by using the Nonlinear Integrated Positive Position Feedback

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Authors' contributions

The sole author designed, analyzed, interpreted and prepared the manuscript.

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Abstract

In this article, the nonlinear integrated positive position feedback (NIPPF) control adds to a nonlinear dynamical system modeled as the well known Duffing oscillators. This control is proposed to mitigate system nonlinear vibrations. The whole system mathematical model is analyzed by applying the multiple time scales perturbation method. The slow-flow modulation equations that govern the oscillation amplitudes of both the main system and controller are derived. The stability of the steady-state solution is presented and studied applying frequency response equations near the simultaneous primary and internal resonance cases. Before and after (NIPPF) control the nonlinear systems' steady-state amplitude are examined, the comparison is made to validate the closeness between the numerical solution and the analytical perturbative one at time-history and frequency response curves.

Keywords: The nonlinear integrated positive position feedback control; modified duffing equation; stability analysis.

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1 Introduction

Many types of controllers are used for suppressing the vibrations of different non-linear dynamical systems such that, negative linear velocity feedback, negative cubic velocity feedback, non-linear saturation controllers (NSC), non-linear Integral Positive Position Feedback Controllers (NIPPF), the Integral resonant controllers (IRC) and time delay control. The technique of multiple time scales used to investigate the micro-beams non-linear vibrations for two different resonance cases (super- harmonic and harmonic resonances). From this investigation, there is a clear effect of the boundary conditions on the micro-beams vibrations [1]. Recently, the vibrations of many vibrating systems [2-8] has been suppressed using different types of control. Because of the time delayed and active controls springiness [9-14] in controlling many vibrating system, many papers used time delay for suppressing the vibrations of non-linear systems. Abdelhafez and Nassar [15], investigated the effectiveness of time delays when the positive position controllers are used for suppressing the vibrations of a self-exited non-linear beam. They notified that, the time margin depends on the overall delays of the system. The authors in [16] investigated the influence of two different delays the first is displacement delay and the second is velocity delay in a cantilever beam. They used the method of multiple scales to determine all super-harmonic and sub-harmonic resonance cases. Since the aim of most studies is to suppress the vibrations, one of the important types of control to vibrating systems is the NIPPF, which, is used as a novel method that merges the characteristics of IRC and PPF methods to manage the oscillatory nonlinear systems. The NIPPF controller has an intelligent result since it decreases the vibration at the correct resonant frequency [17]. Also, a new novel procedure is presented [18] to overcome vibration of the nonlinear oscillatory active structures. For different resonance cases the NIPPF control is applied to reduce the vibrations of duffing oscillator system near primary and super-harmonic resonances [19], through primary and internal resonance [20]. Omidi et al [21,22] presented three kinds of control to suppress the vibrations of vibrating systems such that, the Integral resonant controllers (IRC), PPF controllers and the non-linear Integral Positive Position feedback (NIPPF). The eminent type of decreasing the vibrations is NIPPF type .The NIPPF controller is used for deceasing the vibrations of the model of micro-electro- mechanical system near primary resonance and one-to-one internal resonance [23, 24]. The equations of frequency response are in use to investigate the stability of the obtained solution. The influence of some chosen coefficient is illustrated numerically and analytically. The rapprochement between numeric and analytic solution is offered.

2 System Modeling

Consider the model of micro-electro- mechanical system

$$\begin{aligned}&\ddot{u} + 2\varepsilon\mu_1\dot{u} + \omega_1^2 u + \varepsilon(\alpha_1 u^2 + \alpha_2 u^3) - \varepsilon\alpha(2u + 3u^2 + 4u^3) - \\ &\varepsilon(2u + 3u^2 + 4u^3)(f_1 cos(\Omega t) + f_2 cos(2\Omega t)) - \\ &\varepsilon(\alpha + f_1 cos(\Omega t) + f_2 cos(2\Omega t)) = f_c, 0 < \varepsilon < 1, \end{aligned} \tag{2.1}$$

This model represented the modified Duffing equation subjected to weakly non-linear parametric and external excitations, and described the main motions at time scales of the natural vibrations of the microstructure and fast dynamic at time scales of the high-frequency voltage, μ_1 is the coefficient of viscous damping,ε is a small parameter, ω_1 is linear natural frequency , Ω is the frequency of the external excitation, α is the coefficient of linear term ,α_1, α_2 are the coefficients of the nonlinear terms ,f_1, f_2 are the coefficient of linear and nonlinear parameters excitations, and $f_c(t)$ is the control input. This NIPPF control module is designed in a feedback format for the controller so that it absorbs some of the vibration energy by increasing System damping, compensates for the resonance energy using the application of positive feedback. In order to achieve this goal, The NIPPF controller is described as follows:

$$\ddot{x} + 2\varepsilon\mu_2\omega_2\dot{x} + \omega_2^2 x = \varepsilon\gamma_1 u(t),$$
$$\dot{z} + \sigma z = \varepsilon\gamma_2 u(t), \quad (2.2)$$

with the control law of: $f_c = \lambda_1 x(t) + \lambda_2 z(t)$. so the closed loop system equations are

$$\ddot{u} + 2\varepsilon\mu\dot{u} + \omega_1^2 u + \varepsilon(\alpha_1 u^2 + \alpha_2 u^3) - \varepsilon\alpha(2u + 3u^2 + 4u^3) -$$
$$\varepsilon(2u + 3u^2 + 4u^4)(f_1\cos(\Omega t) + f_2\cos(2\Omega t)) - \varepsilon(\alpha + f_1\cos(\Omega t) + f_2\cos(2\Omega t)) =$$
$$\varepsilon\lambda_1 x(t) + \varepsilon\lambda_2 z(t),$$
$$\ddot{x} + 2\varepsilon\mu_2\omega_2\dot{x} + \omega_2^2 x = \varepsilon\gamma_1 u(t),$$
$$\dot{z} + \sigma z = \varepsilon\gamma_2 u(t), \quad (2.3)$$

where $x(t)$ is the second-order section variable for the NIPPF controller and $z(t)$ is the integrating section variable for the NIPPF controller. μ_2 , ω_2 are the damping factor and internal frequency for the controller, respectively.$\gamma_1 > 0$ and $\gamma_2 > 0$ are the gains of controller, λ_1 is the positive scalar feedback gain of the second- order section, λ_2 is the positive scalar feedback gain of integrating section, σ is the lossy integrator's frequency.

3 Mathematical Analysis

The multiple scales method is applied to get the asymptotic first-order approximate solutions for the system (2.3) which we use the multiscale perturbed method

$$u(T_0, T_1, \varepsilon) = u_0(T_0, T_1) + \varepsilon u_1(T_0, T_1) + O(\varepsilon^2),$$
$$y(T_0, T_1, \varepsilon) = y_0(T_0, T_1) + \varepsilon y_1(T_0, T_1) + O(\varepsilon^2), T_n = \varepsilon^n t, \quad (3.1)$$

where $T_0 = t$ and $T_1 = \varepsilon t$ are the fast and slow time scales, respectively. The time derivatives became

$$\frac{d}{dt} = D_0 + \varepsilon D_1 + ...,$$
$$\frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + ... \quad (3.2)$$

where $D_j = \frac{d}{dT_j}, j = 0, 1$. Substituting (3.1) and (3.2) into (2.3), and equating the coefficients of equal power of ε lead to:

$$O(\varepsilon^0):$$
$$(D_0^2 + \omega_1^2)u_0 = 0,$$
$$(D_0^2 + \omega_2^2)y_0 = 0,$$
$$(D_0 + \sigma)z_0 = \gamma_2 u_0, \quad (3.3)$$

$$O(\varepsilon^1):$$
$$(D_0^2 + \omega_1^2)u_1 = -2D_0D_1u_0 - 2\mu_1 D_0 u_0 - \alpha_1 u_0^2 - \alpha_2 u_0^3 + \alpha(2u_0 + 3u_0^2 + 4u_0^3) +$$
$$(2u_0 + 3u_0^2 + 4u_0^3)(f_1 cos\Omega t + f_2 cos2\Omega t) + \alpha + f_1\cos\Omega t + f_2 \cos 2\Omega t +$$
$$\lambda_1 x_0(t) + \lambda_2 z_0(t),$$
$$(D_0^2 + \omega_2^2)x_1 = -2D_0D_1x_0 - 2\mu_2\omega_2 D_0 x_0 + \gamma_1 u_0,$$
$$(D_0 + \sigma)z_1 = \gamma_2 u_0 - D_1 z_1, \quad (3.4)$$

The solution of system of equations(3.3) are

$$u_0(T_0,T_1) = A_1(T_1)e^{i\omega_1 T_0} + c.c.,$$
$$y_0(T_0,T_1) = A_2(T_1)e^{i\omega_2 T_0} + c.c.$$
$$z_0(T_0,T_1) = A_3(T_1)e^{-\sigma T_0} + \frac{\gamma_2(\sigma - i\omega_1)}{(\omega_1^2+\sigma^2)}A_1(T_1)e^{i\omega_1 T_0}, \tag{3.5}$$

Where A_1, A_2 are unknown complex function in T_1 and c.c. denotes the complex conjugate of the previous terms, insert eq.(3.5) into eq.(3.4) we get

$$\begin{aligned}(D_0^2+\omega_1^2)u_1 = {} & \alpha + A_1\bar{A}_1(6\alpha - 2\alpha_1) + \\ & [-2i\omega_1 D_1A_1 - 2iA_1\mu_1\omega_1 + 2\alpha A_1 + 12\alpha A_1^2\bar{A}_1 - 3\alpha_2 A_1^2\bar{A}_1 + \\ & \frac{\gamma_2(\sigma_i\omega_1)A_1}{(\omega_1^2+\sigma^2)}]e^{i\omega_1 T_0} + (3\alpha-\alpha_1)A_1^2e^{2i\omega_1 T_0} + (4\alpha-\alpha_2)A_1^3e^{3i\omega_1 T_0} + \\ & f_1(0.5+3A_1\bar{A}_1)e^{i\Omega T_0} + f_2(0.5+3A_1\bar{A}_1)e^{2i\Omega T_0} + \\ & 1.5f_1A_1^2e^{i(\Omega+2\omega_1)T_0} + 1.5f_2A_1^2e^{i(2\Omega+2\omega_1)T_0} + \\ & f_1(A_1+6A_1^2\bar{A}_1)e^{i(\Omega+\omega_1)T_0} + f_2(A_1+6A_1^2\bar{A}_1)e^{i(2\Omega+\omega_1)T_0} + \\ & 2f_1A_1^3e^{i(\Omega+3\omega_1)T_0} + 2f_2A_1^3e^{i(2\Omega+3\omega)T_0} + \\ & A_2\lambda_1e^{i\omega_2 T} + A_2\gamma_1e^{i\omega_2 T_0} + A_3\lambda_2e^{-\sigma T_0},\end{aligned} \tag{3.6}$$

$$(D_0^2+\omega_2^2)x_1 = A_1\gamma_1e^{i\omega_1 T_0} - (2iA_2\mu_2\omega_2 + 2iD_1A_2\omega_2)e^{i\omega_2 T_0} + c.c., \tag{3.7}$$

the solutions of equations (3.6),(3.7) after eliminating the secular terms

$$\begin{aligned}u_1 = {} & \alpha + A_1\bar{A}_1(6\alpha-2\alpha_1) + E_1e^{2i\omega_1 T_0} + E_2e^{3i\omega_1 T_0} + E_3e^{i\Omega T_0} + E_4e^{2i\Omega T_0} + \\ & E_5e^{i(\Omega+\omega_1)T_0} + E_6e^{i(2\Omega+\omega_1)T_0} + E_7e^{i(\Omega+2\omega_1)T_0} + \\ & E_8e^{i(\Omega+3\omega_1)T_0} + E_9e^{i(2\Omega+2\omega_1)T_0} + \\ & E_{10}e^{i(2\Omega+3\omega_1)T_0} + E_{11}e^{i\omega_2 T_0} + E_{12}e^{-\sigma T_0} + c.c.\end{aligned} \tag{3.8}$$

$$x_1 = \frac{A_1\gamma_1}{(\omega_2^2-\omega_1^2)}e^{i\omega_2 T_0} + c.c., \tag{3.9}$$

$$\begin{aligned}z_1 = {} & -\frac{i\gamma_2\omega_1(\sigma-i\omega_1)^2}{(\sigma^2+\omega_1^2)^2}e^{i\omega_1 T_0}DA_1 + \alpha + A_1\bar{A}_1(6\alpha-2\alpha_1) + \\ & N_1e^{2i\omega_1 T_0} + N_2e^{3i\omega_1 T_0} + N_3e^{i\Omega T_0} + N_4e^{2i\Omega T_0} + \\ & N_5e^{i(\Omega+\omega_1)T_0} + N_6e^{i(2\Omega+\omega_1)T_0} + N_7e^{i(\Omega+2\omega_1)T_0} + \\ & N_8e^{i(\Omega+3\omega_1)T_0} + N_9e^{i(2\Omega+2\omega_1)T_0} + \\ & N_{10}e^{i(2\Omega+3\omega_1)T_0} + N_{11}e^{i\omega_2 T_0} + c.c.\end{aligned} \tag{3.10}$$

where E_i,i=1,2,....,12 and N_j,j=1,2,...,11 are presented at appendix.

4 Stability Analysis

In this paper, the case of the simultaneous primary and internal resonance ($\Omega = \omega_1, \omega_1 = \omega_2$) which is the worst resonance case, is considered to study the stability of the system of equations (2.3) . Introducing the detuning parameters σ_1 and σ_2 according to:

$$\Omega = \omega_1 + \varepsilon\sigma_1, \omega_2 = \omega_1 + \varepsilon\sigma_2, \tag{4.1}$$

and write

$$\begin{aligned}
&(\Omega - 2\omega_1)T_0 = (\omega_1 + \varepsilon\sigma_1 - 2\omega_1)T_0 = (\varepsilon\sigma_1 - \omega_1)T_0 = -(\omega_1 T_0 - \sigma_1 T_1),\\
&(2\Omega - \omega_1)T_0 = (2\omega_1 + 2\varepsilon\sigma_1 - \omega_1)T_0 = \omega_1 T_0 + 2\sigma_1 T_1,\\
&(2\Omega - 3\omega_1)T_0 = ((2\omega_1 + 2\varepsilon\omega_1 - 3\omega_1)T_0 = -(\omega_1 T_0 - 2\sigma_1 T_1).
\end{aligned} \tag{4.2}$$

Substituting equations (4.1) and (4.2) into equations (3.6) and (3.7) and eliminating the secular terms, leads to the solvability conditions for the first order approximation, hence the following differential equations are obtained:

$$\begin{aligned}
2i\omega_1 D_1 A_1 = &-2iA_1\mu_1\omega_1 + 2\alpha A_1 + 12\alpha A_1^2\bar{A}_1 + (0.5f_1 + 3A_1 f_1\bar{A}_1)e^{i\sigma_1 T_1} +\\
&A_2\lambda_1 e^{i\sigma_2 T_1} + f_2(\bar{A}_1 + 6A_1\bar{A}^2)e^{2i\sigma_1 T_1} + \frac{3f_1 A_1^2 e^{-i\sigma_1 T_1}}{2} - 3A_1^2\alpha_2\bar{A}_1 +\\
&2f_2 A_1^3 e^{-2i\sigma_1 T_1} + \frac{\gamma_2\lambda_2(\sigma - i\omega_1)}{(\omega_1^2 + \sigma^2)}A_1,
\end{aligned} \tag{4.3}$$

$$2i\omega_2 D_1 A_2 = A_1\gamma_1 e^{-i\sigma_2 T_1} - 2iA_2\mu_2\omega_2, \tag{4.4}$$

The solution of equations (4.3) and (4.4) can be analyzed by putting $A_1(T_1), A_2(T_1)$ in polar form,

$$A_1(T_1) = \frac{a_1(T_1)}{2}e^{i\phi_1(T_1)}, A_2(T_1) = \frac{a_2(T_1)}{2}e^{i\phi_2(T_1)} \tag{4.5}$$

$$D_1 A_1 = \frac{1}{2}(\dot{a_1} + ia_1\dot{\phi_1})e^{i\phi_1 T_1}, D_1 A_2 = \frac{1}{2}(\dot{a_2} + ia_2\dot{\phi_2})e^{i\phi_2 T_1}, \tag{4.6}$$

where a_1, a_2 are the amplitudes of steady state, ϕ_1, ϕ_2 are the motions phases. By substituting equations (4.5),(4.6) into equations (4.3) ,(4.4), we get

$$\begin{aligned}
(\dot{a_1} + ia_1\dot{\phi_1}) = &\frac{-i\alpha a_1}{\omega_1} - \mu a_1 - \frac{3i\alpha a_1^3}{2\omega_1} + \frac{3i\alpha_2 a_1^3}{8\omega_1} - \frac{i}{\omega_1}(\frac{1}{2}f_1 + \frac{3}{4}a_1^2 f_1)e^{i(\sigma_1 T_1 - \phi_1)} -\\
&\frac{ia_2\lambda_1}{2\omega_1}e^{i(\sigma_2 T_1 - \phi_1 + \phi_2)} - \frac{i}{2\omega_1}(\frac{f_2 a_1}{2} + \frac{3f_2 a_1^3}{4})e^{2i(\sigma_1 T_1 - \phi_1)} -\\
&\frac{3i}{8\omega_1}f_1 a_1^2 e^{-i(\sigma_1 T_1 - \phi_1)} - \frac{if_2 a_1^3}{4\omega_1}e^{-2i(\sigma_1 T_1 - \phi_1)} - i\frac{\gamma_2\lambda_2(\sigma - i\omega_1)}{\omega_1(\omega_1^2 + \sigma^2)}a_1,
\end{aligned} \tag{4.7}$$

$$(\dot{a_2} + ia_2\dot{\phi_2}) = -a_2\mu_2 - \frac{\gamma_1 a_1 i}{2\omega_2}e^{-i(\sigma_2 T_1 - \phi_1 + \phi_2)}, \tag{4.8}$$

compare the imaginary part and the real terms

$$\begin{aligned}
\dot{a_1} = &-\mu_1 a_1 - \frac{\lambda_2\gamma_2 a_1}{2(\sigma^2 + \omega_1^2)} + \frac{1}{2\omega_1}(a_1 f_2 + \frac{3}{2}a_1^3 f_2)\sin 2\theta_1 + \frac{a_2\lambda_1}{2\omega_1}\sin\theta_2 +\\
&\frac{1}{2\omega_1}(f_1 + \frac{3}{2}a_1^2 f_1)\sin\theta_1 - \frac{3a_1^2 f_1}{8\omega_1}\sin\theta_1 - \frac{f_2 a_1^3}{4\omega_1}\sin 2\theta_1,\\
a_1\dot{\phi_1} = &\frac{-\alpha a_1}{\omega_1} + \frac{\lambda_2\gamma_2\sigma a_1}{2\omega_1(\sigma^2 + \omega_1^2)} - \frac{3\alpha a_1^3}{2\omega_1} + \frac{3\alpha_2 a_1^3}{8\omega_1} - \frac{1}{2\omega_1}(a_1 f_2 + \frac{3}{2}a_1^3 f_2)\cos 2\theta_1 -\\
&\frac{a_2\lambda_1}{2\omega_1}\cos\theta_2 - \frac{1}{2\omega_1}(f_1 + \frac{3}{2}a_1^2 f_1)\cos\theta_1 - \frac{3a_1^2 f_1}{8\omega_1}\cos\theta_1 - \frac{f_2 a_1^3}{4\omega_1}\cos 2\theta_1,
\end{aligned} \tag{4.9}$$

$$\dot{a_2} = -\mu_2 a_2 - \frac{\gamma_1 a_1}{2\omega_2}\sin\,\theta_2,$$
$$a_2\dot{\phi_2} = -\frac{\gamma_1 a_1}{2\omega_2}\cos\,\theta_2, \tag{4.10}$$

where

$$\theta_1 = \sigma_1 T_1 - \phi_1, \theta_2 = \sigma_2 T_1 - \phi_1 + \phi_2, \dot{\theta_1} = \sigma_1 - \dot{\phi_1}, \dot{\theta_2} = \sigma_2 - \dot{\phi_1} + \dot{\phi_2}. \tag{4.11}$$

5 Stability Investigation

The steady-state solution of our dynamical system corresponding to the fixed point of equations (4.9) , (4.10) is obtained when $\dot{a_m} = 0, \dot{\phi_m} = 0, m = 1, 2,$

$$\mu_1 a_1 = -\frac{\lambda_2\gamma_2 a_1}{2(\sigma^2+\omega_1^2)} + \frac{1}{2\omega_1}(a_1 f_2 + \frac{3}{2}a_1^3 f_2)\text{sin}2\theta_1 + \frac{a_2\lambda_1}{2\omega_1}\text{sin}\theta_2 + \frac{1}{2\omega_1}(f_1 + \frac{3}{2}a_1^2 f_1)\text{sin}\theta_1 - \frac{3a_1^2 f_1}{8\omega_1}\text{sin}\theta_1 - \frac{f_2 a_1^3}{4\omega_1}\text{sin}2\theta_1, \tag{5.1}$$

$$\sigma_1 a_1 = \frac{\lambda_2\gamma_2\sigma a_1}{2\omega_1(\sigma^2+\omega_1^2)} - \frac{\alpha a_1}{\omega_1} - \frac{3\alpha a_1^3}{2\omega_1} + \frac{3\alpha_2 a_1^3}{8\omega_1} - \frac{1}{2\omega_1}(a_1 f_2 + \frac{3}{2}a_1^3 f_2)\cos\,2\theta_1 - \frac{a_2\lambda_1}{2\omega_1}\text{cos}\theta_2 - \frac{1}{2\omega_1}(f_1 + \frac{3}{2}a_1^2 f_1)\cos\,\theta_1 - \frac{3a_1^2 f_1}{8\omega_1}\text{cos}\theta_1 - \frac{f_2 a_1^3}{4\omega_1}\text{cos}2\theta_1, \tag{5.2}$$

$$\mu_2 a_2 = -\frac{\gamma_1 a_1}{2\omega_2}\text{sin}\theta_2, \tag{5.3}$$

$$(\sigma_1 - \sigma_2)a_2 = -\frac{\gamma_1 a_1}{2\omega_2}\text{cos}\theta_2. \tag{5.4}$$

From equations (5.1) to (5.4) the amplitude and phase modulating equations take the form

$$\dot{a_1} = -\mu_1 a_1 - \frac{\lambda_2\gamma_2 a_1}{2(\sigma^2+\omega_1^2)} + \frac{1}{2\omega_1}(a_1 f_2 + \frac{3}{2}a_1^3 f_2)\text{sin}2\theta_1 + \frac{a_2\lambda_1}{2\omega_1}\text{sin}\theta_2 + \frac{1}{2\omega_1}(f_1 + \frac{3}{2}a_1^2 f_1)\text{sin}\theta_1 - \frac{3a_1^2 f_1}{8\omega_1}\text{sin}\theta_1 - \frac{f_2 a_1^3}{4\omega_1}\text{sin}2\theta_1, \tag{5.5}$$

$$\dot{\theta_1} = \sigma_1 + \frac{\lambda_2\gamma_2\sigma}{2\omega_1(\sigma^2+\omega_1^2)} + \frac{\alpha}{\omega_1} + \frac{3\alpha a_1^2}{2\omega_1} - \frac{3\alpha_2 a_1^2}{8\omega_1} + \frac{1}{2\omega_1}(f_2 + \frac{3}{2}a_1^2 f_2)\cos\,2\theta_1 + \frac{a_2\lambda_1}{2a_1\omega_1}\text{cos}\theta_2 + \frac{1}{2\omega_1}(\frac{f_1}{a_1} + \frac{3}{2}a_1 f_1)\cos\,\theta_1 + \frac{3a_1 f_1}{8\omega_1}\text{cos}\theta_1 + \frac{f_2 a_1^2}{4\omega_1}\text{cos}2\theta_1, \tag{5.6}$$

$$\dot{a_2} = -\mu_2\omega_1 a_2 - \frac{\gamma_1 a_1}{2\omega_1}\sin\,\theta_2, \tag{5.7}$$

$$\dot{\theta_2} = \sigma_2 + \frac{\lambda_2\gamma_2\sigma}{2\omega_1(\sigma^2+\omega_1^2)} + \frac{\alpha}{\omega_1} + \frac{3\alpha a_1^2}{2\omega_1} - \frac{3\alpha_2 a_1^2}{8\omega_1} + \frac{1}{2\omega_1}(f_2 + \frac{3}{2}a_1^2 f_2)\cos\,2\theta_1 + \frac{a_2\lambda_1}{2a_1\omega_1}\cos\,\theta_2 + \frac{1}{2\omega_1}(\frac{f_1}{a_1} + \frac{3}{2}a_1 f_1)\text{cos}\theta_1 + \frac{3a_1 f_1}{8\omega_1}\text{cos}\theta_1 - \frac{\gamma_1 a_1}{2a_2\omega_2}\text{cos}\theta_2 + \frac{f_2 a_1^2}{4\omega_1}\text{cos}2\theta_1, \tag{5.8}$$

To determine the stability of the nonlinear solution, one lets

$$a_1 = a_{10} + a_{11}, a_2 = a_{20} + a_{21}, \theta_1 = \theta_{10} + \theta_{11}, \theta_2 = \theta_{20} + \theta_{21}, \tag{5.9}$$

where a_{m0}, θ_{m0} are the solutions of equations (5.5)-(5.8) and a_{m1}, θ_{m1} are perturbations which are assumed to be small compared to a_{m0}, θ_{m0} . Substituting equation (5.9) into equations (5.5)-(5.8) and keeping only the linear terms in a_{m1}, θ_{m1} , we obtain that

$$\dot{a_{11}} = [-\mu_1 + \frac{3a_{10}f_1\sin\theta_{10}}{4\omega_1} + \frac{(f_2 + 3a_{10}^2 f_2)\sin2\theta_{10}}{2\omega_1} - \frac{\lambda_2\gamma_2}{2\sigma_2 + \omega_1^2}]a_{11} + [\frac{\cos\theta_{10}(4f_1 + 3a_{10}^2 f_1)}{8\omega_1} + \frac{(4f_2 a_{10} + 3a_{10}^3 f_2)\cos2\theta_{10}}{4\omega_1}]\theta_{11} + [\frac{\sin\theta_{20}\lambda_1}{2\omega_1}]a_{21} + [\frac{a_{20}\lambda_1\cos\theta_{20}}{2\omega_1}]\theta_{21}, \tag{5.10}$$

$$\dot{\theta_{11}} = [\frac{\sigma_1}{a_{10}} + \frac{\alpha}{a_{10}\omega_1} + \frac{3a_{10}\alpha}{\omega_1} - \frac{3a_{10}\alpha_2}{4\omega_1} + \frac{9f_1\cos\theta_{10}}{8\omega_1} + \frac{2f_2 a_{10}}{\omega_1}\cos2\theta_{10} + \frac{\lambda_2\gamma_2\sigma}{2a_{10}\omega_1(\sigma^2 + \omega_1^2)}]a_{11} + [-(\frac{f_1}{2\omega_1 a_{10}} + \frac{9f_1 a_{10}}{8\omega_1})\sin\theta_{10} - \frac{(4f_2 + 9a_{10}^2 f_2)}{4\omega_1}\sin2\theta_{10}]\theta_{11} + [\frac{\lambda_1\cos\theta_{20}}{2a_{10}\omega_1}]a_{21} - [\frac{\lambda_1 a_{20}\sin\theta_{20}}{2a_{10}\omega_1}]\theta_{21}, \tag{5.11}$$

$$\dot{a_{21}} = [-\frac{\gamma_1\sin(\theta_{20})}{2\omega_2}]a_{11} + 0\theta_{11} - \mu_2 a_{21} - [\frac{\gamma_1 a_{10}}{2\omega_2}\cos\theta_{20}]\theta_{21}, \tag{5.12}$$

$$\dot{\theta_{21}} = [-\frac{\gamma_1}{2a_{20}\omega_2}\cos\theta_{20} + \frac{\sigma_1}{a_{10}} + \frac{\alpha}{a_{10}\omega_1} + \frac{3a_{10}\alpha}{\omega_1} - \frac{3a_{10}\alpha_2}{4\omega_1} + \frac{9f_1\cos\theta_{10}}{8\omega_1} + \frac{2f_2 a_{10}\cos2\theta_{10}}{\omega_1} + \frac{\lambda_2\gamma_2\sigma}{2a_{10}\omega_1(\sigma^2 + \omega_1^2)}]a_{11} + [-(\frac{f_1}{2\omega_1 a_{10}} + \frac{9f_1 a_{10}}{8\omega_1})\sin\theta_{10} - \frac{(4f_2 + 9a_{10}^2 f_2)}{4\omega_1}\sin2\theta_{10})]\theta_{11} + [(\frac{\sigma_2 - \sigma_1}{a_{20}} + \frac{\lambda_1\cos\theta_{20}}{2a_{10}\omega_1})a_{21} + (\frac{\lambda_1 a_{10}}{2a_{20}\omega_2} - \frac{\lambda_1 a_{20}}{2a_{10}\omega_1})\sin\theta_{20}]\theta_{21}, \tag{5.13}$$

The following linear system is topologically equivalent to the nonlinear system given by Equations from (5.10) to (5.13) as long as the eigenvalues are hyperbolic

$$\begin{pmatrix} \dot{a_{11}} \\ \dot{\theta_{11}} \\ \dot{a_{21}} \\ \dot{\theta_{21}} \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ r_{31} & 0 & r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & r_{44} \end{pmatrix} \begin{pmatrix} a_{11} \\ \theta_{11} \\ a_{21} \\ \theta_{21} \end{pmatrix} \tag{5.14}$$

The eigenvalues of the Jacobian matrix can be obtained by resolving the following determinant

$$\begin{pmatrix} \lambda - r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & \lambda - r_{22} & r_{23} & r_{24} \\ r_{31} & 0 & \lambda - r_{33} & r_{34} \\ r_{41} & r_{42} & r_{43} & \lambda - r_{44} \end{pmatrix} = 0 \tag{5.15}$$

the values of eigenvalues are the roots of the following polynomial

$$\lambda^4 + R_1\lambda^3 + R_2\lambda^2 + R_3\lambda + R_4 = 0, \tag{5.16}$$

According to Routh–Hurwitz criterion, the necessary and sufficient conditions for the system stability are: $R_1 > 0, R_1R_2 - R_3 > 0, R_3(R_1R_2 - R_3) - R_1^2R_4 > 0, R_4 > 0.$

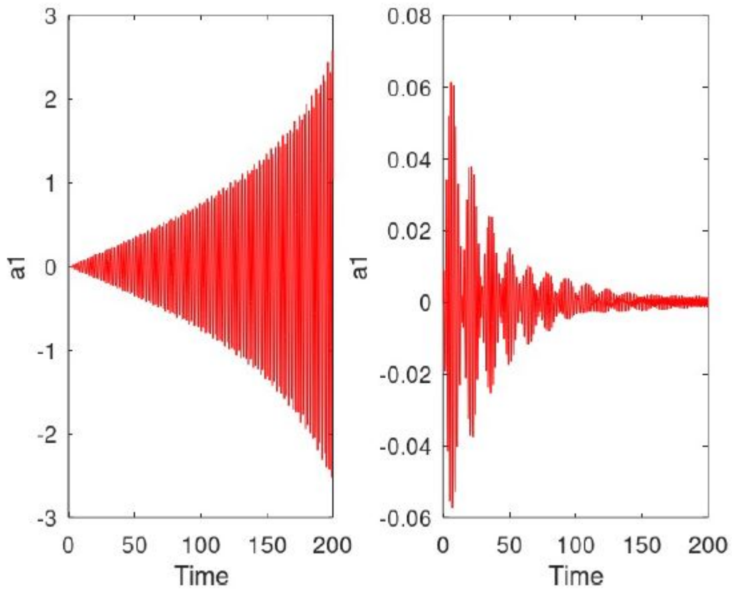

Fig. 1. The vibration amplitudes of main system a without control and b with NIPPF control

6 Time history

we simulated numerically equation (2.1) which introduced the nonlinear dynamical model without and with involved NIPPF control to show the reduce of vibration after adding this control. After inserting the values of parameters as$\mu = 0.01, \alpha = 0.01, \alpha_1 = 1.5, \alpha_2 = 0.02, \gamma_1 = \gamma_2 = 1.5, \omega_1 = \omega_2 = \Omega = 3.5, \xi = 0.003, f_1 = 0.05, f_2 = 0.5, \sigma = 3, \lambda_1 = \lambda_1 = 1.5$ the time history can be illustrated as in Fig.(1) a and b which represents the uncontrolled amplitude time history at primary resonance of the main model and the time histories of both controlled amplitude of the main model with NIPPF. We study the effects of different parameters by solving the frequency response equations (5.1) - (5.4). The results are illustrated graphically in Figs. (2 to 14). From the obtained figures, the steady state amplitudes a_1 and a_2 are presented against detuning parameters σ_1, σ_2 for the selected practical case $(a_1 \neq 0, a_2 \neq 0)$. The following curves represent the frequency response of the system with NIPPF control, where Fig. (a) shows the frequency response curves for the system) and Fig. (b) shows the frequency-response curves for NIPPF controller. At $\sigma_1 = 0$ the minimum steady-state amplitude a_1 is zero. Figs. (2), (3) shows that the steady state amplitudes for both the main system and the NIPPF controller are increased according to the increasing values of the excitation forces amplitudes f_1, f_2. The controlled main system amplitudes are inversely proportional to the gains of the control λ_1, λ_2 as shown in Figs. 4, 5 and Fig. 6 shown that for increasing γ_1 the controlled main system amplitudes is decreasing and wider. , for increase γ_2 the values of amplitude a_1, a_2 increase. Figure (8) shows that for increasing values of the damping coefficients μ_1 both the main system and the controller are decreasing. Fig.(9) represent the affect of the damping coefficient of the (NIPPF) controller for increasing μ_2 the amplitude of the main system and control are decreasing.

Fig. (10) show that the increase of linear natural frequency ω_1 makes a decrease in the amplitude of the main system and the vibration reduction frequency bandwidth of the control for the amplitude of the main system a_1 is wider. The controlled main system amplitudes are inversely proportional to the lossy integrator's frequency , the coefficient linear and nonlinear term α, α_2 as shown in Figs.11, 12, 13. The fig.(14)shows that when taking different values of the internal detuning parameter σ_2 the shape of the frequency response curves for both the main system and the controller are affected by different values, for example when $\sigma_2 = 0.5$ the minimum steady state amplitude for the main system occurs when $\sigma_1 = 0.5$, for $\sigma_2 = 0$ the minimum steady state amplitude for the main system occurs when $\sigma_1 = 0$, and for $\sigma_2 = 0.5$ The steady-state widening of the main system of the small candle occurs when $\sigma_1 = 0.5$ So, at $\sigma_1 = \sigma_2$ the lower main system steady-state amplitude occurs.

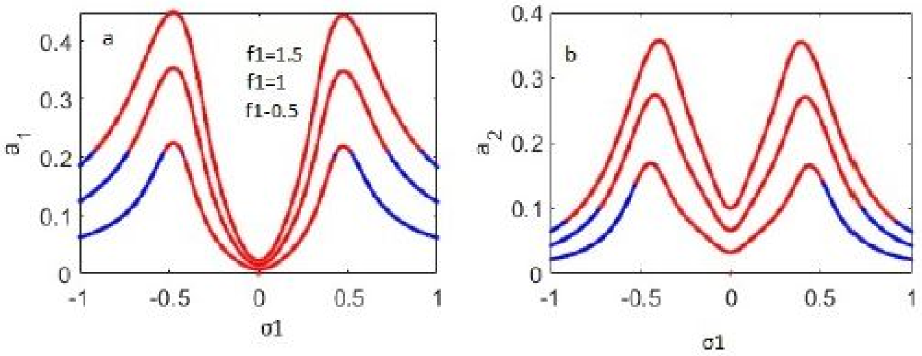

Fig. 2. Effect of the linear external excitation force f_1 on: a the main system (a_1), and b the control (a_2)

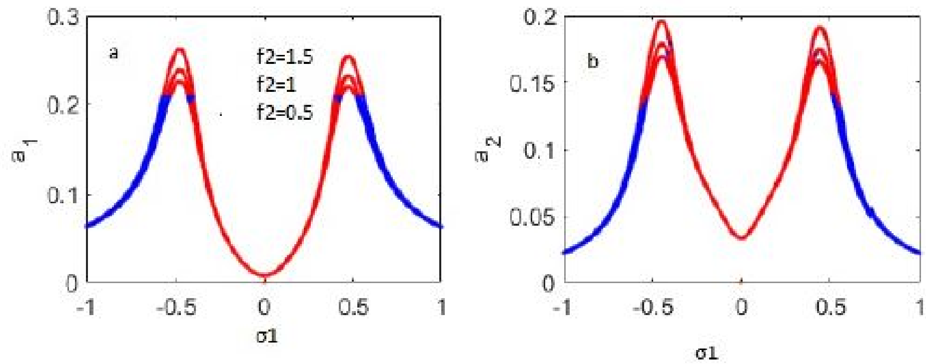

Fig. 3. Effect of the nonlinear external excitation force f_2 on: a the main system (a_1), and b the control (a_2)

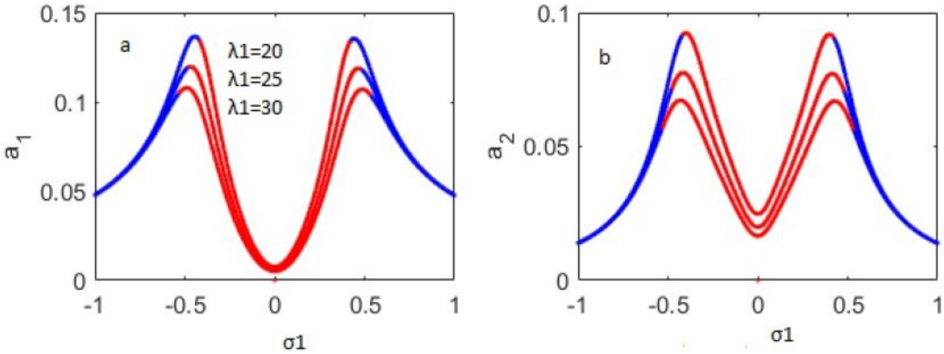

Fig. 4. The feedback gain λ_1 effectiveness on : a main system and b on the NIPPF control

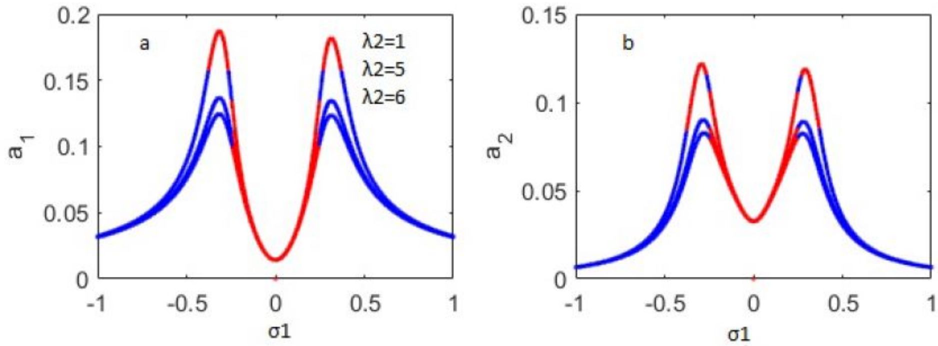

Fig. 5. The feedback gain λ_2 effectiveness on: a main system and b on the NIPPF control

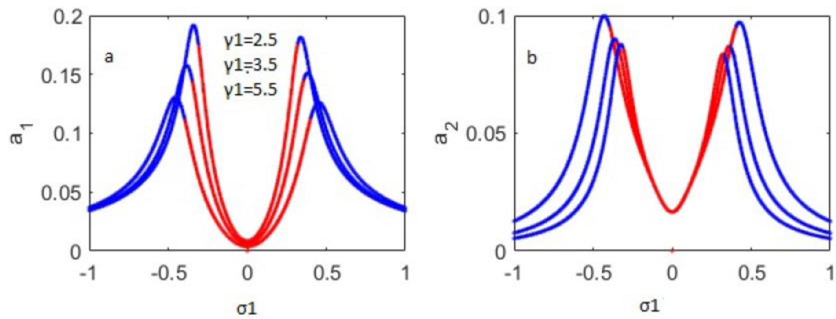

Fig. 6. The feedback gain γ_1 effectiveness on : a main system and b on the NIPPF control

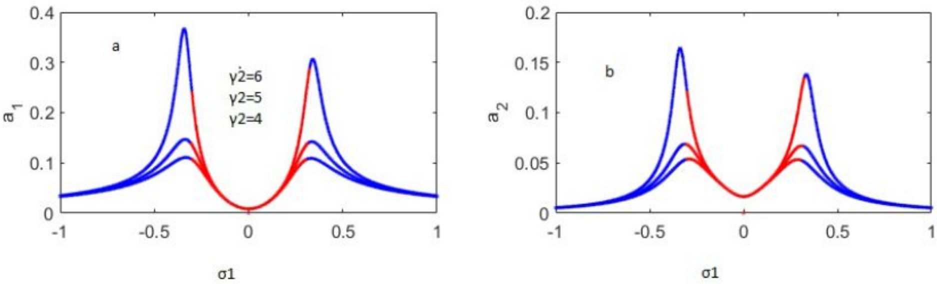

Fig. 7. The feedback gain γ_2 effectiveness on: a main system and b on the NIPPF control

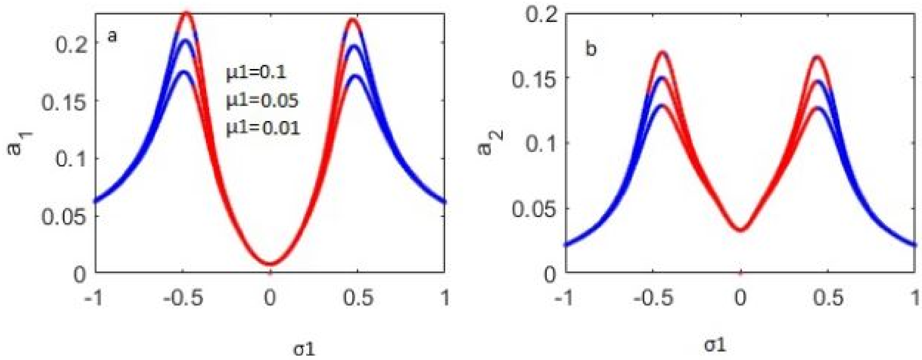

Fig. 8. Effect of μ_1 is the coefficient of viscous damping on the amplitudes of main system and NIPPF control

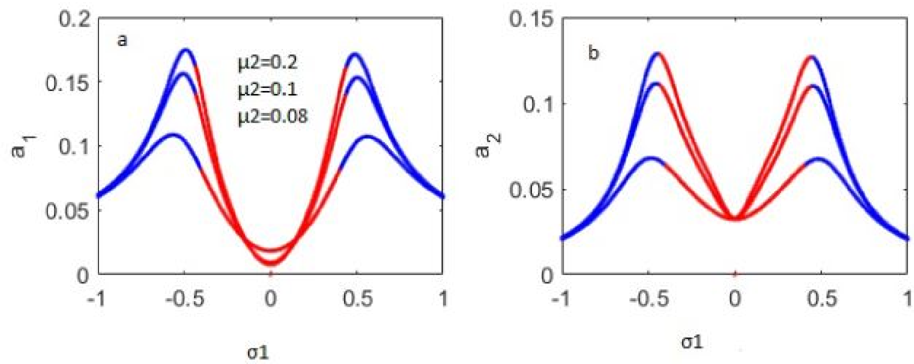

Fig. 9. Effect of μ_2 is the coefficient of viscous damping on the amplitudes of main system and NIPPF control

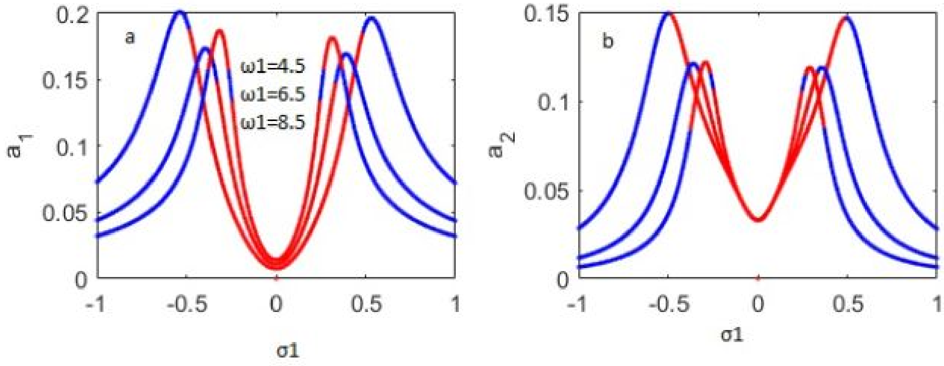

Fig. 10. Effect of linear natural frequency on the amplitudes of main system and NIPPF control

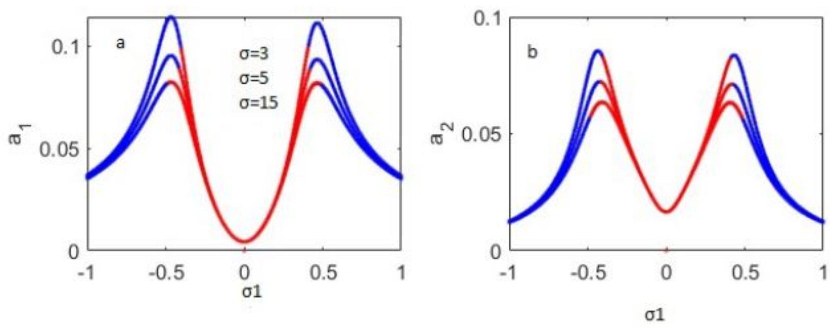

Fig. 11. Effect of the lossy integrator's frequency σ on the amplitudes of main system and NIPPF control

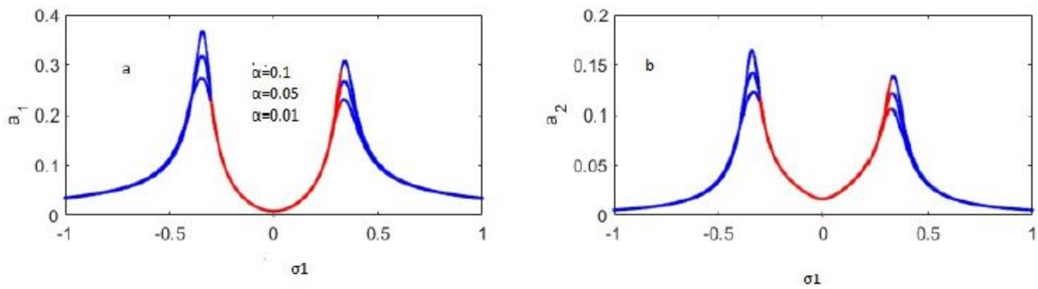

Fig. 12. Effect of the coefficient of linear term α on the amplitudes of main system and NIPPF control

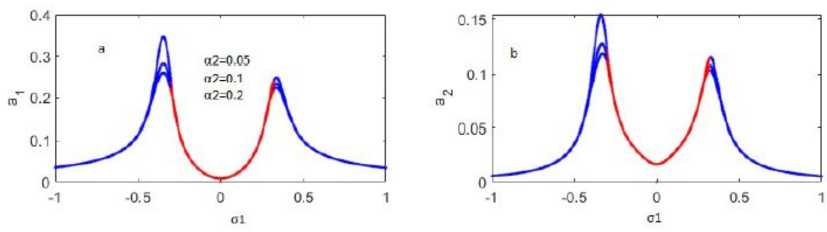

Fig. 13. Effect of the coefficient nonlinear term α_2 on the amplitudes of main system and NIPPF control

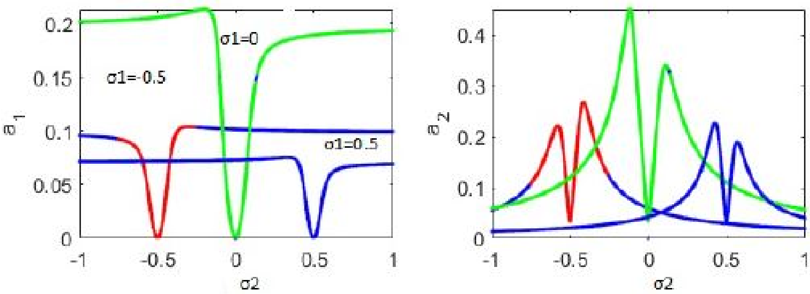

Fig.14. The effect of damping parameterσ_2 on both the amplitudes of main system and NIPPF control

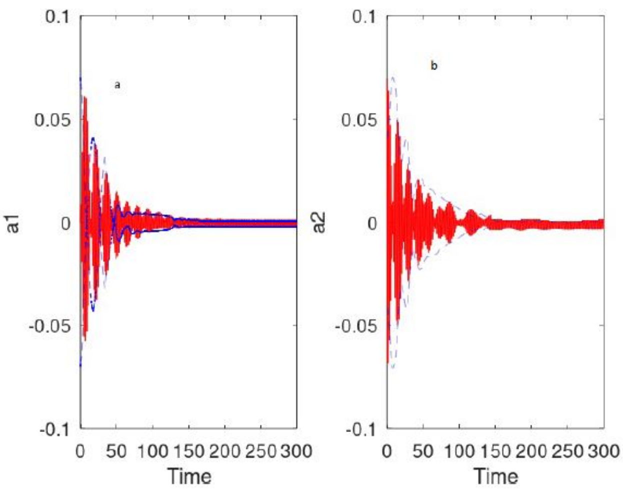

Fig. 15. Comparison between the numerical solution and the perturbation analysis of closed loop for the amplitudes a_1, a_2 of main system and NIPPF control

7 Comparison between Analytical and Numerical Solutions

Figure (15) represents the comparison between the numerical solution of equations (2.3) and the analytical solution The solution given by equations (5.1-5.4) for the modified Duffing equation with the NIPPF controller for chosen values of system parameters. The dashed lines show the analytical solution and represent the continuous lines numerical solution.

8 Conclusions

In this paper, the modified duffing equation is studied with NIPPF control to reduce the vibration. We use the simultaneous primary and internal resonance case by the method of multiple scales. The stability of the system under the simultaneous resonances is studied to drive the frequency response equations. The effects of the different parameters of the system and the controller are studied numerically. The numerical results are focused on both the effects of different parameters and the response of the system.

Competing Interests

Author has declared that no competing interests exist.

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APPENDIX

$$E_1 = \frac{(3\alpha A_1^2 - A_1^2\alpha_1)}{-3\omega_1^2}, E_2 = \frac{(4\alpha A_1^3 - A_1^3\alpha_2)}{-8\omega_1^2}, E_3 = \frac{(0.5f_1 + 3A_1 f_1 \bar{A}_1)}{(\omega_1^2 - \Omega_1^2)},$$
$$E_4 = \frac{(0.5f_2 + 3A_1 f_2 \bar{A}_1)}{(\omega_1^2 - 4\Omega^2)}, E_5 = \frac{(A_1 f_1 + 6A_1^2 f_1 \bar{A}_1)}{(\omega_1^2 - (\Omega + \omega_1)^2)}, E_6 = \frac{(A_1 f_2 + 6A_1^2 f_2 \bar{A}_1)}{(\omega_1^2 - (2\Omega + \omega_1)^2)},$$
$$E_7 = \frac{1.5A_1^2 f_1}{(\omega_1^2 - (\Omega + 2\omega_1)^2)}, E_8 = \frac{2A_1^3 f_1}{(\omega_1^2 - (\Omega + 3\omega_1)^2)}, E_9 = \frac{1.5A_1^2 f_2}{(\omega_1^2 - (2\Omega + 2\omega_1)^2)},$$
$$E_{10} = \frac{2A_1^3 f_2}{(\omega_1^2 - (2\Omega + 3\omega_1)^2)}, E_{11} = \frac{A_2(\lambda_1 - \gamma_1)}{\omega_1^2 - \omega_2^2}, E_{12} = \frac{A_3\lambda_2}{\omega_1^2 + \sigma^2},$$
$$N_1 = \frac{\gamma_2 E_1}{2i\omega_1 + \sigma}, N_2 = \frac{\gamma_2 E_2}{3i\omega_1 + \sigma}, N_3 = \frac{\gamma_2 E_3}{i\Omega + \sigma}, N_4 = \frac{\gamma_2 E_4}{2i\Omega + \sigma},$$
$$N_5 = \frac{\gamma_2 E_5}{i(\Omega + \omega_1) + \sigma}, N_6 = \frac{\gamma_2 E_6}{i(2\Omega + \omega_1) + \sigma}, N_7 = \frac{\gamma_2 E_7}{i(\Omega + 2\omega_1) + \sigma},$$
$$N_8 = \frac{\gamma_2 E_9}{i(\Omega + 3\omega_1) + \sigma}, N_9 = \frac{\gamma_2 E_8}{i(2\Omega + 2\omega_1) + \sigma}, N_{10} = \frac{\gamma_2 E_{10}}{i(2\Omega + 3\omega_1) + \sigma},$$
$$N_{11} = \frac{\gamma_2 E_{11}}{i\omega_2 + \sigma}, \quad (8.1)$$

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