



Comparing Calibration Product Type Estimators of Population Mean In Stratified Sampling under Two Constraints Using Different Distance Measures

D. N. Ojua^{1*}, J. A. Abuchu², E. O. Ojua³ and E. I. Enang¹

¹Department of Statistics, University of Calabar, Calabar, Nigeria.

²Department of Mathematics, University of Calabar, Calabar, Nigeria.

³Department of Plant Science and Biotechnology, University of Nigeria, Nsukka, Nigeria.

Authors' contributions

This work was carried out in collaboration among all authors. Authors DNO and EIE designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author JAA managed the literature searches. Author EOO managed the analyses of the study and wrote the first draft of the manuscript. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JAMCS/2021/v36i930397

Editor(s):

(1) Dr. Burcu Gurbuz, Uskudar University, Turkey.

Reviewers:

(1) Abdelhadi Assir, Hassan First University of Settat, Morocco.

(2) Shpetim Rexhepi, State University of Tetovo, Macedonia.

(3) Ette Harrison Etuk, Rivers State University, Nigeria.

Complete Peer review History: <https://www.sdiarticle4.com/review-history/65900>

Received: 10 January 2021

Accepted: 13 March 2021

Published: 21 October 2021

Original Research Article

Abstract

Calibration approach adjusts the original design weights by incorporating an auxiliary variable into it, to make the estimator be in the form of a regression estimator. This method was employed to propose calibration product type estimators using three distance measures namely; chi-square distance measure, the minimum entropy distance measure and the modified chi-square distance measure using double constraints. The estimators of variances of the proposed estimators were also obtained. An empirical study to ascertain the performance of these estimators using a secondary data set and simulated data under underlying distributional assumptions of Gamma, Normal and Exponential distributions with varying sample sizes of 10%, 15%, 20% and 25% were carried out. The result with the real life data showed that the calibration product type estimator \bar{y}_{pc42} from chi-square distance measure estimated the population mean with minimum bias than \bar{y}_{pc5} and \bar{y}_{pc6} obtained from the other distance measures. The result from real life data

*Corresponding author: E-mail: dorisojua@gmail.com;

also revealed that the estimator \bar{y}_{pc41} obtained from chi-square distance measure under two constraints was more efficient than the other three estimators. The result from simulation studies showed that the proposed calibration product type estimators outperform the conventional product type estimator in term of efficiency, consistency and reliability under the Gamma and Exponential distributions with the exponential distribution taking the lead. The conventional product type estimator however was found to be better under normal distribution. It was also observed that as sample size increases there was no significant change in the performance of these proposed estimators which justifies the preference with small sample size.

Keywords: Finite population; auxiliary variable; stratified sampling; calibration estimators; population mean.

1 Introduction

The regression estimator is an unbiased estimator of the population parameter which uses information on the auxiliary variable x which is correlated with the study variable y . The ratio estimator is used when the variables x is positively correlated with y , while the product type estimator is preferred when the variate x is negatively correlated with y . Robson [1], Murthy [2] and Perri [3] had established that both the ratio and product type estimator are good estimators of the population parameters if the regression line is a straight line and passes through the origin. However in many practical situations the regression line does not pass through the origin and in such situations the ratio and product estimators do not perform as well as the regression estimator [4].

Calibration approach adjusts the original design weight by incorporating an auxiliary variable into it, and makes the estimator to be in the form of a regression estimator. This method has been used by several authors such as Deville and Sarndal [5], Tracy, Singh and Arnab [6], Clement and Enang [7], Koyuncu and Kadilar [8], Singh and Arnab [9] amongst others to propose some estimators which have a form of the regression estimator. But so far in sampling literature, the product type estimator has not yet been written in form of a regression estimator. This work seeks to use the calibration approach to rewrite the conventional product type estimator in form of a regression estimator.

1.1 Definition of terms

\bar{X}_h is the population mean of the auxiliary variable
 \bar{x}_h is the sample mean of the auxiliary variable
 \bar{Y}_h is the population mean of the variable of interest
 \bar{y}_h is the sample mean of the variable of interest
 S_{hy}^2 is the population variance of the variable of interest
 s_{hy}^2 is the sample variance of the variable of interest
 S_{hx}^2 is the population variance of the auxiliary variable
 s_{hx}^2 is the sample variance of the auxiliary variable
 S_{hxy} is the covariance between the auxiliary variable and variable of interest
 ρ_{xy} is the correlation between the variable of interest and the auxiliary variable
 N is the population size
 n is the sample size
 N_h is the stratum population size
 n_h is the stratum sample size
 Q_h is a positive constant
MSE is the mean square error

1.2 Percentage average relative efficiency (% \overline{RE})

The relative efficiency of two procedures is given by the ratio of their efficiencies and is often defined using variance or mean square error. This shall be used to measure the average efficiency of each proposed estimator. It can be computed as:

$$\% \overline{RE}(\hat{y}_{pcp}) = \left\{ \frac{\sqrt{MSE(\hat{y}_p)}}{\sqrt{MSE(\hat{y}_{pcp})}} \right\} \times 100 \tag{1}$$

Where

$$\overline{MSE}(\hat{y}_{pcp}) = \frac{1}{h} \sum_{h=1}^H MSE(\hat{y}_{pcp}) \tag{2}$$

It should be noted that a $\% \overline{RE}(\hat{y}_{pcp})$ of value greater than 100 predicts a relative increase in efficiency of the proposed estimator, while a $\% \overline{RE}(\hat{y}_{pcp})$ of value less than 100 indicates a loss in efficiency of the proposed estimator.

1.3 Percentage average absolute relative bias $\%(\overline{ARB})$

If \hat{y}_{pcp} , then, for each stratum $h = 1, 2, \dots, L$, the relative bias is given by:

$$RB(\hat{y}_{pcp}) = \frac{1}{R} \sum_{r=1}^R \left(\frac{\hat{y}_{pcp}}{\bar{y}_p} - 1 \right) \tag{3}$$

and the percentage average absolute relative bias $\%(\overline{ARB})$ is computed as

$$\% \overline{ARB}(\hat{y}_{pcp}) = \left\{ \frac{1}{L} \sum_{h=1}^L ARB(\hat{y}_{pcp}) \right\} \times 100 \tag{4}$$

where

$$ARB(\hat{y}_{pcp}) = \left| \frac{1}{R} \sum_{r=1}^R \left(\frac{\hat{y}_{pcp}}{\bar{y}_p} - 1 \right) \right| \tag{5}$$

Where R is the number of runs.

1.4 Average coefficient of variation (\overline{CV})

This measure shall be used to measure the reliability of the proposed estimators compared to the conventional product type estimator in stratified sampling. The percentage average coefficient of variation of \hat{y}_{pcp} is given as:

$$\% \overline{CV}(\hat{y}_{pcp}) = \left\{ \frac{1}{L} \sum_{h=1}^L CV(\hat{y}_{pcp}) \right\} \times 100 \tag{6}$$

Where

$$CV(\hat{y}_{pcp}) = \frac{\sqrt{MSE(\hat{y}_{pcp})}}{\bar{y}_p}$$

The interpretation is that, high values of $\%CV(\hat{y}_{pcp})$ indicate unreliable estimates while low value predicts reliable estimates.

2 Proposed Estimators

Theorem 2.1: Given the product type estimator

$$\bar{y}_{ps} = \sum_{h=1}^L W_h (\bar{x}_h \bar{y}_h) / (\bar{X}_h)$$

a calibration product type estimator \bar{y}_{pcp4} for population mean \bar{Y} given as

$$\begin{aligned} \bar{y}_{pcp4} &= \sum_{h=1}^L W_h (\bar{x}_h \bar{y}_h) / (\bar{X}_h) \\ &+ \left(\left(\sum_{h=1}^L W_h Q_h \bar{x}_h^2 \bar{y}_h / \bar{X}_h \right) \left(\sum_{h=1}^L W_h Q_h \right) - \left(\sum_{h=1}^L W_h Q_h \bar{x}_h \bar{y}_h / \bar{X}_h \right) \left(\sum_{h=1}^L W_h Q_h \bar{x}_h \right) \right) / \left(\sum_{h=1}^L W_h Q_h \right) \left(\sum_{h=1}^L W_h Q_h \bar{x}_h^2 \right) \\ &- \left(\sum_{h=1}^L W_h Q_h \bar{x}_h \right)^2 \left(\bar{X} - \sum_{h=1}^L W_h \bar{x}_h \right) \end{aligned}$$

can be obtained by

$$MinD = \sum_{h=1}^L \frac{(\gamma_{h4} - W_h)^2}{W_h Q_h}$$

s.t.

$$\sum_{h=1}^L \gamma_{h4} = \sum_{h=1}^L W_h$$

$$\sum_{h=1}^L \gamma_{h4} \bar{x}_h = \bar{X}$$

Where the constraints states that the sum of the design weight is equal to the sum of the calibrated weight and the sum of the calibrated weight multiplied by the strata mean of the auxiliary variable equals the population mean of the auxiliary variable.

Proof: Given the product type estimator, an estimator as defined as

$$\bar{y}_{pcp4} = \sum_{h=1}^L \gamma_{h4} \frac{\bar{x}_h \bar{y}_h}{\bar{X}_h} \tag{7}$$

where the weight γ_{h4} are chosen such that the distance measure

$$\sum_{h=1}^L \frac{(\gamma_{h4} - W_h)^2}{W_h Q_h}$$

Is minimized subject to the constraint

$$\sum_{h=1}^L \gamma_{h4} = \sum_{h=1}^L W_h \tag{8}$$

and

$$\sum_{h=1}^L \gamma_{h4} \bar{x}_h = \bar{X} \tag{9}$$

Combining the distance measure, (8) and (9) gives the optimization function

$$\varphi(\gamma_{h4}, \lambda_{41}, \lambda_{42}) = \sum_{h=1}^L \frac{(\gamma_{h4} - W_h)^2}{W_h Q_h} - 2\lambda_{41} (\sum_{h=1}^L \gamma_{h4} - \sum_{h=1}^L W_h) - 2\lambda_{42} (\sum_{h=1}^L \gamma_{h4} \bar{x}_h - \bar{X}) \tag{10}$$

Differentiating equation (10) partially with respect to γ_{h4} , λ_{41} and λ_{42} and equating to zero gives

$$\gamma_{h4} = W_h [1 + \lambda_{41} Q_h + \lambda_{42} Q_h \bar{x}_h] \tag{11}$$

$$\lambda_{41} = \frac{-(\sum_{h=1}^L W_h Q_h \bar{x}_h)(\bar{X} - \sum_{h=1}^L W_h \bar{x}_h)}{(\sum_{h=1}^L W_h Q_h)(\sum_{h=1}^L W_h Q_h \bar{x}_h^2) - (\sum_{h=1}^L W_h Q_h \bar{x}_h)^2} \tag{12}$$

and

$$\lambda_{42} = \frac{(\sum_{h=1}^L W_h Q_h)(\bar{X} - \sum_{h=1}^L W_h \bar{x}_h)}{(\sum_{h=1}^L W_h Q_h)(\sum_{h=1}^L W_h Q_h \bar{x}_h^2) - (\sum_{h=1}^L W_h Q_h \bar{x}_h)^2} \tag{13}$$

substituting (12) and (13) into (11) gives

$$\gamma_{h4} = W_h + \frac{(W_h Q_h \bar{x}_h)(\sum_{h=1}^L W_h Q_h) - (W_h Q_h)(\sum_{h=1}^L W_h Q_h \bar{x}_h)}{(\sum_{h=1}^L W_h Q_h)(\sum_{h=1}^L W_h Q_h \bar{x}_h^2) - (\sum_{h=1}^L W_h Q_h \bar{x}_h)^2} (\bar{X} - \sum_{h=1}^L W_h \bar{x}_h) \tag{14}$$

substituting equation (14) into equation (7) we obtain

$$\bar{y}_{pcp4} = \sum_{h=1}^L \frac{W_h \bar{x}_h \bar{y}_h}{\bar{X}_h} + \frac{\left(\sum_{h=1}^L \frac{W_h Q_h \bar{x}_h^2 \bar{y}_h}{\bar{X}_h}\right)(\sum_{h=1}^L W_h Q_h) - \left(\sum_{h=1}^L \frac{W_h Q_h \bar{x}_h \bar{y}_h}{\bar{X}_h}\right)(\sum_{h=1}^L W_h Q_h \bar{x}_h)}{(\sum_{h=1}^L W_h Q_h)(\sum_{h=1}^L W_h Q_h \bar{x}_h^2) - (\sum_{h=1}^L W_h Q_h \bar{x}_h)^2} (\bar{X} - \sum_{h=1}^L W_h \bar{x}_h) \tag{15}$$

Which is the proposed calibration product type estimator for population mean \bar{Y} in stratified random sampling. The proposed estimator is in form of a regression equation with $\sum_{h=1}^L \frac{W_h \bar{x}_h \bar{y}_h}{\bar{X}_h}$ as the intercept and $\frac{\left(\sum_{h=1}^L \frac{W_h Q_h \bar{x}_h^2 \bar{y}_h}{\bar{X}_h}\right)(\sum_{h=1}^L W_h Q_h) - \left(\sum_{h=1}^L \frac{W_h Q_h \bar{x}_h \bar{y}_h}{\bar{X}_h}\right)(\sum_{h=1}^L W_h Q_h \bar{x}_h)}{(\sum_{h=1}^L W_h Q_h)(\sum_{h=1}^L W_h Q_h \bar{x}_h^2) - (\sum_{h=1}^L W_h Q_h \bar{x}_h)^2}$ as the slop.

Substituting $Q_h = 1$ and $Q_h = \frac{1}{\bar{x}_h}$ in (15) gives

$$\bar{y}_{pcp41} = \sum_{h=1}^L \frac{W_h \bar{x}_h \bar{y}_h}{\bar{X}_h} + \frac{\left(\sum_{h=1}^L \frac{W_h \bar{x}_h^2 \bar{y}_h}{\bar{X}_h}\right)(\sum_{h=1}^L W_h) - \left(\sum_{h=1}^L \frac{W_h \bar{x}_h \bar{y}_h}{\bar{X}_h}\right)(\sum_{h=1}^L W_h \bar{x}_h)}{(\sum_{h=1}^L W_h)(\sum_{h=1}^L W_h \bar{x}_h^2) - (\sum_{h=1}^L W_h \bar{x}_h)^2} (\bar{X} - \sum_{h=1}^L W_h \bar{x}_h) \tag{16}$$

and

$$\bar{y}_{pcp42} = \sum_{h=1}^L \frac{W_h \bar{x}_h \bar{y}_h}{\bar{X}_h} + \frac{\left(\sum_{h=1}^L \frac{W_h \bar{x}_h \bar{y}_h}{\bar{X}_h}\right)\left(\sum_{h=1}^L W_h \frac{1}{\bar{x}_h}\right) - \left(\sum_{h=1}^L \frac{W_h \bar{y}_h}{\bar{X}_h}\right)(\sum_{h=1}^L W_h)}{\left(\sum_{h=1}^L W_h \frac{1}{\bar{x}_h}\right)(\sum_{h=1}^L W_h \bar{x}_h) - (\sum_{h=1}^L W_h)^2} (\bar{X} - \sum_{h=1}^L W_h \bar{x}_h) \tag{17}$$

Where equation (16) and (17) are called the regression and ratio type calibration product type estimator for population mean for stratified sampling respectively.

Theorem 2.2: Given the product type estimator, a calibration product type estimator \bar{y}_{pcp5} for population mean \bar{Y} given as

$$\bar{y}_{pcp5} = \sum_{h=1}^L \frac{W_h \bar{x}_h \bar{y}_h}{\bar{X}_h}$$

can be obtained by

$$MinD = \sum_{h=1}^L \frac{1}{Q_h} \left\{ \gamma_{h5} \log \left(\frac{\gamma_{h5}}{W_h} \right) - \gamma_{h5} - W_h \right\}$$

s.t.

$$\sum_{h=1}^L \gamma_{h5} = \sum_{h=1}^L W_h$$

$$\sum_{h=1}^L \gamma_{h5} \bar{x}_h = \bar{X}$$

Proof: Given the product type estimator, an estimator as defined as

$$\bar{y}_{pcp5} = \sum_{h=1}^L \gamma_{h5} \frac{\bar{x}_h \bar{y}_h}{\bar{X}_h} \tag{18}$$

where the weights γ_{h5} are chosen such that the distance measure

$$\sum_{h=1}^L \frac{1}{Q_h} \left\{ \gamma_{h5} \log \left(\frac{\gamma_{h5}}{W_h} \right) - \gamma_{h5} - W_h \right\}$$

is minimized subject to constraints

$$\sum_{h=1}^L \gamma_{h5} = \sum_{h=1}^L W_h$$

and

$$\sum_{h=1}^L \gamma_{h5} \bar{x}_h = \bar{X}$$

By combining the distance measures and the constraints gives the optimization function

$$\varphi(\gamma_{h5}, \lambda_{51}, \lambda_{52}) = \sum_{h=1}^L \frac{1}{Q_h} \left\{ \gamma_{h5} \log \left(\frac{\gamma_{h5}}{W_h} \right) - \gamma_{h5} - W_h \right\} - 2\lambda_{51} (\sum_{h=1}^L \gamma_{h5} - \sum_{h=1}^L W_h) - 2\lambda_{52} (\sum_{h=1}^L \gamma_{h5} \bar{x}_h - \bar{X}) \tag{19}$$

Differentiating equation (19) partially with respect to $\gamma_{h5}, \lambda_{51}$ and λ_{52} , and equating to zero gives

$$\gamma_{h5} = W_h \exp[\lambda_{51} Q_h + \lambda_{52} Q_h \bar{x}_h] \tag{20}$$

and taking $\lambda_{51} = 1$ arbitrarily gives

$$\lambda_{52} = \frac{-\sum_{h=1}^L Q_h}{\sum_{h=1}^L Q_h \bar{x}_h}$$

substituting λ_{51} and λ_{52} into (20) gives

$$\gamma_{h5} = W_h \exp \left[Q_h - \frac{(Q_h \bar{x}_h) \sum_{h=1}^L Q_h}{\sum_{h=1}^L Q_h \bar{x}_h} \right] \tag{21}$$

substituting equation (21) into equation (18), we obtain

$$\bar{y}_{pcp5} = \sum_{h=1}^L \frac{W_h \bar{x}_h \bar{y}_h}{\bar{X}_h} \tag{22}$$

In this case, we observed that when λ_{51} is taken to be one arbitrarily and appropriate substitution done in the calibration equation, the calibration equation reduces to the conventional product estimator in stratified

sampling. Hence the proposed calibration product type estimator is equal to the conventional product type estimator.

Theorem 2.3: Calibration product type estimator \bar{y}_{pcp6} for population mean \bar{Y} can be obtained from the product type estimator by

$$MinD = \sum_{h=1}^L \frac{(\gamma_{h6} - W_h)^2}{\gamma_{h6} Q_h}$$

s.t.

$$\sum_{h=1}^L \gamma_{h6} = \sum_{h=1}^L W_h$$

$$\sum_{h=1}^L \gamma_{h6} \bar{x}_h = \bar{X}$$

given as

$$\bar{y}_{pcp6} = \sum_{h=1}^L \frac{W_h \bar{x}_h \bar{y}_h}{\bar{x}_h} \left(L + \frac{(\sum_{h=1}^L Q_h)(\sum_{h=1}^L W_h^2 Q_h \bar{x}_h) - (\sum_{h=1}^L Q_h \bar{x}_h)(\sum_{h=1}^L W_h^2 Q_h)}{\bar{X}^2 [(\sum_{h=1}^L W_h^2 Q_h)(\sum_{h=1}^L Q_h \bar{x}_h) - (\sum_{h=1}^L Q_h)(\sum_{h=1}^L W_h^2 Q_h \bar{x}_h)]} (\bar{X}^2 - \sum_{h=1}^L W_h^2 \bar{x}_h^2) \right)^{-1/2}$$

Proof: Given the product type estimator, a calibration estimator defined as

$$\bar{y}_{pcp6} = \sum_{h=1}^L \gamma_{h6} \frac{\bar{x}_h \bar{y}_h}{\bar{X}_h} \tag{23}$$

where the weights γ_{h6} are chosen such that the distance measure

$$\sum_{h=1}^L \frac{(\gamma_{h6} - W_h)^2}{\gamma_{h6} Q_h}$$

is minimized subject to constraints

$$\sum_{h=1}^L \gamma_{h6} = \sum_{h=1}^L W_h$$

and

$$\sum_{h=1}^L \gamma_{h6} \bar{x}_h = \bar{X}$$

Then, by combining the distance measure and the constraints gives the optimization function

$$\varphi(\gamma_{h6}, \lambda_{61}, \lambda_{62}) = \sum_{h=1}^L \frac{(\gamma_{h6} - W_h)^2}{\gamma_{h6} Q_h} - 2\lambda_{61} (\sum_{h=1}^L \gamma_{h6} - \sum_{h=1}^L W_h) - 2\lambda_{62} (\sum_{h=1}^L \gamma_{h6} \bar{x}_h - \bar{X}) \tag{24}$$

Differentiating equation (24) partially with respect to $\gamma_{h6}, \lambda_{61}$ and λ_{62} , and equating to zero gives

$$\gamma_{h6} = \frac{W_h}{[1 - 2\lambda_{61} Q_h - 2\lambda_{62} Q_h \bar{x}_h]^2} \tag{25}$$

$$\lambda_{61} = \frac{-(\sum_{h=1}^L W_h^2 Q_h \bar{x}_h)(\bar{X}^2 - \sum_{h=1}^L W_h^2 \bar{x}_h^2)}{2\bar{X}^2 [(\sum_{h=1}^L W_h^2 Q_h)(\sum_{h=1}^L Q_h \bar{x}_h) - (\sum_{h=1}^L Q_h)(\sum_{h=1}^L W_h^2 Q_h \bar{x}_h)]} \tag{26}$$

and

$$\lambda_{62} = \frac{(\sum_{h=1}^L W_h^2 Q_h)(\bar{X}^2 - \sum_{h=1}^L W_h^2 \bar{x}_h^2)}{2\bar{X}^2[(\sum_{h=1}^L W_h^2 Q_h)(\sum_{h=1}^L Q_h \bar{x}_h) - (\sum_{h=1}^L Q_h)(\sum_{h=1}^L W_h^2 Q_h \bar{x}_h)]} \tag{27}$$

Substituting (26) and (27) into (25) gives

$$\gamma_{h6} = W_h \left[1 + \frac{(Q_h)(\sum_{h=1}^L W_h^2 Q_h \bar{x}_h) - (Q_h \bar{x}_h)(\sum_{h=1}^L W_h^2 Q_h)}{2\bar{X}^2[(\sum_{h=1}^L W_h^2 Q_h)(\sum_{h=1}^L Q_h \bar{x}_h) - (\sum_{h=1}^L Q_h)(\sum_{h=1}^L W_h^2 Q_h \bar{x}_h)]} (\bar{X}^2 - \sum_{h=1}^L W_h^2 \bar{x}_h^2) \right]^{\frac{1}{2}} \tag{28}$$

substituting equation (28) into equation (23) gives

$$\bar{y}_{pcp6} = \sum_{h=1}^L \frac{W_h \bar{x}_h \bar{y}_h}{\bar{x}_h} \left(L + \frac{(\sum_{h=1}^L Q_h)(\sum_{h=1}^L W_h^2 Q_h \bar{x}_h) - (\sum_{h=1}^L Q_h \bar{x}_h)(\sum_{h=1}^L W_h^2 Q_h)}{\bar{X}^2[(\sum_{h=1}^L W_h^2 Q_h)(\sum_{h=1}^L Q_h \bar{x}_h) - (\sum_{h=1}^L Q_h)(\sum_{h=1}^L W_h^2 Q_h \bar{x}_h)]} (\bar{X}^2 - \sum_{h=1}^L W_h^2 \bar{x}_h^2) \right)^{-1/2} \tag{29}$$

Which is the proposed calibration product type estimator for population mean \bar{Y} in stratified random sampling, as required to prove.

Substituting $Q_h = 1$ and $Q_h = \frac{1}{\bar{x}_h}$ in (29) gives

$$\bar{y}_{pcp61} = \sum_{h=1}^L \frac{W_h \bar{x}_h \bar{y}_h}{\bar{x}_h} \left(L + \frac{L(\sum_{h=1}^L W_h^2 \bar{x}_h) - (\sum_{h=1}^L \bar{x}_h)(\sum_{h=1}^L W_h^2)}{\bar{X}^2[(\sum_{h=1}^L W_h^2)(\sum_{h=1}^L \bar{x}_h) - L(\sum_{h=1}^L W_h^2 \bar{x}_h)]} (\bar{X}^2 - \sum_{h=1}^L W_h^2 \bar{x}_h^2) \right)^{-1/2} \tag{30}$$

and

$$\bar{y}_{pcp62} = \sum_{h=1}^L \frac{W_h \bar{x}_h \bar{y}_h}{\bar{x}_h} \left(L + \frac{(\sum_{h=1}^L \frac{1}{\bar{x}_h})(\sum_{h=1}^L W_h^2) - L(\sum_{h=1}^L W_h^2 \frac{1}{\bar{x}_h})}{\bar{X}^2[L(\sum_{h=1}^L W_h^2 \frac{1}{\bar{x}_h}) - (\sum_{h=1}^L \frac{1}{\bar{x}_h})(\sum_{h=1}^L W_h^2)]} (\bar{X}^2 - \sum_{h=1}^L W_h^2 \bar{x}_h^2) \right)^{-1/2} \tag{31}$$

Equation (30) and (31) is the regression and ratio calibration product type estimator for population mean for stratified sampling respectively.

3 Variance Estimators of the Proposed Estimators

Theorem 3.1: Given the product type variance estimator, its weight can be adjusted by

$$MinD = \sum_{h=1}^L \frac{(\omega_{h4}^0 - D_h)^2}{D_h Q_h}$$

s.t.

$$\sum_{h=1}^L \omega_{h4}^0 = \sum_{h=1}^L D_h$$

$$\sum_{h=1}^L \omega_{h4}^0 S_{hx}^2 = V(\bar{x}_{st})$$

to obtain the calibration product type variance estimator $\hat{V}(\bar{y}_{pcp4})$ for population mean \bar{Y} given as

$$\hat{V}(\bar{y}_{pcp4}) = \sum_{h=1}^L \frac{D_h \gamma_{h4}^2}{W_h^2} S_p + \frac{\left(\sum_{h=1}^L \frac{D_h Q_h \gamma_{h4}^2 S_p S_{hx}^2}{W_h^2}\right) \left(\sum_{h=1}^L D_h Q_h\right) - \left(\sum_{h=1}^L \frac{D_h Q_h \gamma_{h4}^2 S_p S_{hx}^2}{W_h^2}\right) \left(\sum_{h=1}^L D_h Q_h S_{hx}^2\right)}{\left(\sum_{h=1}^L D_h Q_h\right) \left(\sum_{h=1}^L D_h Q_h S_{hx}^4\right) - \left(\sum_{h=1}^L D_h Q_h S_{hx}^2\right)^2} \left(V(\bar{x}_{st}) - \hat{v}(\bar{x}_{st})\right)$$

Proof: Given the product type variance estimator, a calibration variance estimator given as

$$\hat{V}(\bar{y}_{pcp4}) = \sum_{h=1}^L \frac{\omega_{h4}^o \gamma_{h4}^2}{W_h^2} S_p \tag{32}$$

where the weights ω_{h4}^o , are chosen such that the distance measure

$$\sum_{h=1}^L \frac{(\omega_{h4}^o - D_h)^2}{D_h Q_h}$$

is minimized subject to the constraint

$$\sum_{h=1}^L \omega_{h4}^o = \sum_{h=1}^L D_h \tag{33}$$

and

$$\sum_{h=1}^L \omega_{h4}^o S_{hx}^2 = V(\bar{x}_{st}) \tag{34}$$

Combining the distance measure and equation (27) and (28), it gives the optimization function

$$\varphi(\omega_{h4}^o, \lambda_{411}, \lambda_{422}) = \sum_{h=1}^L \frac{(\omega_{h4}^o - D_h)^2}{D_h Q_h} - 2\lambda_{411} \left(\sum_{h=1}^L \omega_{h4}^o - \sum_{h=1}^L D_h\right) - 2\lambda_{422} \left(\sum_{h=1}^L \omega_{h4}^o S_{hx}^2 - V(\bar{x}_{st})\right) \tag{35}$$

Differentiating equation (35) partially with respect to ω_{h4}^o , λ_{411} and λ_{422} and equating to zero gives

$$\omega_{h4}^o = D_h [1 + \lambda_{411} Q_h + \lambda_{422} Q_h S_{hx}^2] \tag{36}$$

$$\lambda_{411} = \frac{-\left(\sum_{h=1}^L D_h Q_h S_{hx}^2\right) \left(V(\bar{x}_{st}) - \hat{v}(\bar{x}_{st})\right)}{\left(\sum_{h=1}^L D_h Q_h\right) \left(\sum_{h=1}^L D_h Q_h S_{hx}^4\right) - \left(\sum_{h=1}^L D_h Q_h S_{hx}^2\right)^2} \tag{37}$$

and

$$\lambda_{422} = \frac{\left(\sum_{h=1}^L D_h Q_h\right) \left(V(\bar{x}_{st}) - \hat{v}(\bar{x}_{st})\right)}{\left(\sum_{h=1}^L D_h Q_h\right) \left(\sum_{h=1}^L D_h Q_h S_{hx}^4\right) - \left(\sum_{h=1}^L D_h Q_h S_{hx}^2\right)^2} \tag{38}$$

Substituting (37) and (38) into (36) gives

$$\omega_{h4}^o = D_h + \frac{(D_h Q_h S_{hx}^2) \left(\sum_{h=1}^L D_h Q_h\right) - (D_h Q_h) \left(\sum_{h=1}^L D_h Q_h S_{hx}^2\right)}{\left(\sum_{h=1}^L D_h Q_h\right) \left(\sum_{h=1}^L D_h Q_h S_{hx}^4\right) - \left(\sum_{h=1}^L D_h Q_h S_{hx}^2\right)^2} \left(V(\bar{x}_{st}) - \hat{v}(\bar{x}_{st})\right) \tag{39}$$

Substituting (39) into (32) we obtain

$$\hat{V}(\bar{y}_{pcp4}) = \sum_{h=1}^L \frac{D_h \gamma_{h4}^2}{W_h^2} S_p + \frac{\left(\sum_{h=1}^L \frac{D_h Q_h \gamma_{h4}^2 S_p S_{hx}^2}{W_h^2}\right) \left(\sum_{h=1}^L D_h Q_h\right) - \left(\sum_{h=1}^L \frac{D_h Q_h \gamma_{h4}^2 S_p S_{hx}^2}{W_h^2}\right) \left(\sum_{h=1}^L D_h Q_h S_{hx}^2\right)}{\left(\sum_{h=1}^L D_h Q_h\right) \left(\sum_{h=1}^L D_h Q_h S_{hx}^4\right) - \left(\sum_{h=1}^L D_h Q_h S_{hx}^2\right)^2} \left(V(\bar{x}_{st}) - \hat{v}(\bar{x}_{st})\right) \tag{40}$$

which is the proposed calibration product type variance estimator for population mean \bar{Y} in stratified random sampling as required to prove.

Substituting $Q_h = 1$ and $Q_h = \frac{1}{\bar{x}_h}$ in (40) gives

$$\hat{V}(\bar{y}_{pcp41}) = \sum_{h=1}^L \frac{D_h \gamma_{h41}^2}{W_h^2} S_p + \frac{\left(\sum_{h=1}^L \frac{D_h \gamma_{h41}^2 S_p S_{hx}^2}{W_h^2} \right) \left(\sum_{h=1}^L D_h \right) - \left(\sum_{h=1}^L \frac{D_h \gamma_{h41} S_p S_{hx}^2}{W_h^2} \right) \left(\sum_{h=1}^L D_h S_{hx}^2 \right)}{\left(\sum_{h=1}^L D_h \right) \left(\sum_{h=1}^L D_h S_{hx}^4 \right) - \left(\sum_{h=1}^L D_h S_{hx}^2 \right)^2} (V(\bar{x}_{st}) - \hat{v}(\bar{x}_{st})) \tag{41}$$

Which is the regression calibration variance estimator for population mean \bar{Y} in stratified random sampling.

and

$$\hat{V}(\bar{y}_{pcp42}) = \sum_{h=1}^L \frac{D_h \gamma_{h42}^2}{W_h^2} S_p + \frac{\left(\sum_{h=1}^L \frac{D_h \frac{1}{\bar{x}_h} \gamma_{h42}^2 S_p S_{hx}^2}{W_h^2} \right) \left(\sum_{h=1}^L D_h \right) - \left(\sum_{h=1}^L \frac{D_h \frac{1}{\bar{x}_h} \gamma_{h42} S_p S_{hx}^2}{W_h^2} \right) \left(\sum_{h=1}^L D_h \frac{1}{\bar{x}_h} S_{hx}^2 \right)}{\left(\sum_{h=1}^L D_h \frac{1}{\bar{x}_h} \right) \left(\sum_{h=1}^L D_h \frac{1}{\bar{x}_h} S_{hx}^4 \right) - \left(\sum_{h=1}^L D_h \frac{1}{\bar{x}_h} S_{hx}^2 \right)^2} (V(\bar{x}_{st}) - \hat{v}(\bar{x}_{st})) \tag{42}$$

Which is the ratio calibration variance estimator for population mean \bar{Y} in stratified random sampling.

Theorem 3.2: Given the product type variance estimator, its weight can be adjusted by

$$MinD = \sum_{h=1}^L \frac{1}{Q_h} \left\{ \omega_{h5}^o \log \left(\frac{\omega_{h5}^o}{D_h} \right) - \omega_{h5}^o - D_h \right\}$$

s.t.

$$\sum_{h=1}^L \omega_{h5}^o = \sum_{h=1}^L D_h$$

$$\sum_{h=1}^L \omega_{h5}^o S_{hx}^2 = V(\bar{x}_{st})$$

to obtain the calibration product type variance estimator $\hat{V}(\bar{y}_{pcp5})$ for population mean \bar{Y} given as

$$\hat{V}(\bar{y}_{pcp5}) = \sum_{h=1}^L \frac{D_h \gamma_{h5}^2}{W_h^2} S_p$$

Proof: Rewriting the variance of the product type estimator as

$$\hat{V}(\bar{y}_{pcp5}) = \sum_{h=1}^L \frac{\omega_{h5}^o \gamma_{h5}^2}{W_h^2} S_p \tag{43}$$

Where the weights ω_{h5}^o , are chosen such that the distance measure

$$\sum_{h=1}^L \frac{1}{Q_h} \left\{ \omega_{h5}^o \log \left(\frac{\omega_{h5}^o}{D_h} \right) - \omega_{h5}^o - D_h \right\}$$

is minimized subject to the constraints

$$\sum_{h=1}^L \omega_{h5}^o = \sum_{h=1}^L D_h$$

and

$$\sum_{h=1}^L \omega_{h5}^o S_{hx}^2 = V(\bar{x}_{st})$$

Then by combining the distance measure and the constraints gives the optimization function

$$\varphi(\omega_{h5}^o, \lambda_{511}, \lambda_{522}) = \sum_{h=1}^L \frac{1}{Q_h} \left\{ \omega_{h5}^o \log\left(\frac{\omega_{h5}^o}{D_h}\right) - \omega_{h5}^o - D_h \right\} - \lambda_{511} \left(\sum_{h=1}^L \omega_{h5}^o - \sum_{h=1}^L D_h \right) - \lambda_{522} \left(\sum_{h=1}^L \omega_{h5}^o S_{hx}^2 - V(\bar{x}_{st}) \right) \tag{44}$$

Differentiating equation (44) partially with respect to ω_{h5}^o , λ_{511} and λ_{522} and equating to zero gives

$$\omega_{h5}^o = D_h \exp[1 + \lambda_{511} Q_h + \lambda_{522} Q_h S_{hx}^2] \tag{45}$$

$$\lambda_{511} = 1$$

and

$$\lambda_{522} = \frac{-(\sum_{h=1}^L Q_h)}{(\sum_{h=1}^L Q_h S_{hx}^2)}$$

Substituting for λ_{511} and λ_{522} in (45) gives

$$\omega_{h5}^o = D_h \exp \left[1 + Q_h - \frac{(\sum_{h=1}^L Q_h)}{(\sum_{h=1}^L Q_h S_{hx}^2)} Q_h S_{hx}^2 \right] \tag{46}$$

Substituting (46) into (43) we obtain

$$\hat{V}(\bar{y}_{pcp5}) = \sum_{h=1}^L \frac{D_h \gamma_{h5}^2}{W_h^2} S_p \tag{47}$$

Which is the proposed calibration product type variance estimator for population mean \bar{Y} in stratified random sampling as required to prove.

Theorem 3.3: Given the product type variance estimator, a calibration product type variance estimator $\hat{V}(\bar{y}_{pcp6})$ for population mean \bar{Y} given as

$$\hat{V}(\bar{y}_{pcp6}) = \sum_{h=1}^L \frac{D_h \gamma_{h6}^2}{W_h^2} S_p \left(L + \frac{(\sum_{h=1}^L Q_h)(\sum_{h=1}^L D_h^2 Q_h S_{hx}^2) - (\sum_{h=1}^L Q_h S_{hx}^2)}{(V(\bar{x}_{st}))^2 [(\sum_{h=1}^L D_h^2 Q_h)(\sum_{h=1}^L Q_h S_{hx}^2) - (\sum_{h=1}^L Q_h)(\sum_{h=1}^L D_h^2 Q_h S_{hx}^2)]} \left((V(\bar{x}_{st}))^2 - \sum_{h=1}^L (D_h S_{hx}^2)^2 \right)^{-1/2} \right)$$

can be obtained by

$$MinD = \sum_{h=1}^L \frac{1}{Q_h} \left\{ \omega_{h5}^o \log\left(\frac{\omega_{h5}^o}{D_h}\right) - \omega_{h5}^o - D_h \right\}$$

s.t.

$$\sum_{h=1}^L \omega_{h6}^o = \sum_{h=1}^L D_h$$

$$\sum_{h=1}^L \omega_{h6}^o S_{hx}^2 = V(\bar{x}_{st}).$$

Proof: Given the product type variance estimator, a calibration estimator defined as

$$\hat{V}(\bar{y}_{pcp6}) = \sum_{h=1}^L \frac{\omega_{h6}^o \gamma_{h6}^2}{W_h^2} S_p \tag{48}$$

Where the weights ω_{h6}^o , are chosen such that the distance measure

$$\sum_{h=1}^L \frac{(\omega_{h6}^o - D_h)^2}{\omega_{h6}^o Q_h}$$

is minimized subject to the calibration equation

$$\sum_{h=1}^L \omega_{h6}^o = \sum_{h=1}^L D_h$$

and

$$\sum_{h=1}^L \omega_{h6}^o S_{hx}^2 = V(\bar{x}_{st})$$

By combining the distance measure and the constraints gives the optimization function.

$$\begin{aligned} \varphi(\omega_{h6}^o, \lambda_{611}, \lambda_{622}) &= \sum_{h=1}^L \frac{(\omega_{h6}^o - D_h)^2}{\omega_{h6}^o Q_h} - 2\lambda_{611} (\sum_{h=1}^L \omega_{h6}^o - \sum_{h=1}^L D_h) - \\ &2\lambda_{622} (\sum_{h=1}^L \omega_{h6}^o S_{hx}^2 - V(\bar{x}_{st})) \end{aligned} \tag{49}$$

Differentiating equation (44) partially with respect to ω_{h6}^o , λ_{611} and λ_{622} and equating to zero gives

$$\omega_{h6}^o = \frac{D_h}{[1 - 2\lambda_{611} Q_h - 2\lambda_{622} Q_h S_{hx}^2]^{\frac{1}{2}}} \tag{50}$$

$$\lambda_{611} = \frac{-(\sum_{h=1}^L D_h^2 Q_h S_{hx}^2) (V(\bar{x}_{st}))^2 - \sum_{h=1}^L (D_h S_{hx}^2)^2}{2(V(\bar{x}_{st}))^2 [(\sum_{h=1}^L D_h^2 Q_h)(\sum_{h=1}^L Q_h S_{hx}^2) - (\sum_{h=1}^L Q_h)(\sum_{h=1}^L D_h^2 Q_h S_{hx}^2)]} \tag{51}$$

and

$$\lambda_{622} = \frac{(\sum_{h=1}^L D_h^2 Q_h) (V(\bar{x}_{st}))^2 - \sum_{h=1}^L (D_h S_{hx}^2)^2}{2(V(\bar{x}_{st}))^2 [(\sum_{h=1}^L D_h^2 Q_h)(\sum_{h=1}^L Q_h S_{hx}^2) - (\sum_{h=1}^L Q_h)(\sum_{h=1}^L D_h^2 Q_h S_{hx}^2)]} \tag{52}$$

Substituting (51) and (52) into (50) gives

$$\begin{aligned} \omega_{h6}^o &= D_h \left(1 + \frac{(Q_h)(\sum_{h=1}^L D_h^2 Q_h S_{hx}^2) - (Q_h S_{hx}^2)(\sum_{h=1}^L D_h^2 Q_h)}{(V(\bar{x}_{st}))^2 [(\sum_{h=1}^L D_h^2 Q_h)(\sum_{h=1}^L Q_h S_{hx}^2) - (\sum_{h=1}^L Q_h)(\sum_{h=1}^L D_h^2 Q_h S_{hx}^2)]} \left((V(\bar{x}_{st}))^2 - \right. \right. \\ &\left. \left. \sum_{h=1}^L (D_h S_{hx}^2)^2 \right) \right)^{-1/2} \end{aligned} \tag{53}$$

Substituting (53) into (48) gives

$$\begin{aligned} \hat{V}(\bar{y}_{pcp6}) &= \sum_{h=1}^L \frac{D_h \gamma_{h6}^2}{W_h^2} S_p \left(L + \right. \\ &\left. \frac{(\sum_{h=1}^L Q_h)(\sum_{h=1}^L D_h^2 Q_h S_{hx}^2) - (\sum_{h=1}^L (Q_h S_{hx}^2))}{(V(\bar{x}_{st}))^2 [(\sum_{h=1}^L D_h^2 Q_h)(\sum_{h=1}^L Q_h S_{hx}^2) - (\sum_{h=1}^L Q_h)(\sum_{h=1}^L D_h^2 Q_h S_{hx}^2)]} \left((V(\bar{x}_{st}))^2 - \sum_{h=1}^L (D_h S_{hx}^2)^2 \right) \right)^{-1/2} \end{aligned} \tag{54}$$

Which is the proposed calibration product type variance estimator for population mean \bar{Y} in stratified random sampling as required to prove.

Substituting $Q_h = 1$ and $Q_h = \frac{1}{\bar{x}_h}$ in (49) gives

$$\hat{V}(\bar{y}_{pcp61}) = \sum_{h=1}^L \frac{D_h Y_{h61}^2}{W_h^2} S_p \left(L + \frac{L(\sum_{h=1}^L D_h^2 S_{hx}^2) - (\sum_{h=1}^L (S_{hx}^2))}{(V(\bar{x}_{st}))^2 [(\sum_{h=1}^L D_h^2)(\sum_{h=1}^L S_{hx}^2) - L(\sum_{h=1}^L D_h^2 S_{hx}^2)]} \left((V(\bar{x}_{st}))^2 - \sum_{h=1}^L (D_h S_{hx}^2)^2 \right)^{-1/2} \right) \tag{55}$$

and

$$\hat{V}(\bar{y}_{pcp62}) = \sum_{h=1}^L \frac{D_h Y_{h62}^2}{W_h^2} S_p \left(L + \frac{(\sum_{h=1}^L \frac{1}{\bar{x}_h})(\sum_{h=1}^L D_h^2 \frac{1}{\bar{x}_h} S_{hx}^2) - (\sum_{h=1}^L (\frac{1}{\bar{x}_h} S_{hx}^2))}{(V(\bar{x}_{st}))^2 [(\sum_{h=1}^L D_h^2 \frac{1}{\bar{x}_h})(\sum_{h=1}^L \frac{1}{\bar{x}_h} S_{hx}^2) - (\sum_{h=1}^L \frac{1}{\bar{x}_h})(\sum_{h=1}^L D_h^2 \frac{1}{\bar{x}_h} S_{hx}^2)]} \left((V(\bar{x}_{st}))^2 - \sum_{h=1}^L (D_h S_{hx}^2)^2 \right)^{-1/2} \right) \tag{56}$$

4 Numerical Illustration

In this section empirical evaluation of the proposed calibration estimators is done using stimulated data set with underlying distributional assumption of Normal, Gamma and Exponential and real – life data set from a secondary source to authenticate the result of our study.

4.1 Empirical evaluation of estimators using real-life data

In this section estimate of the mean fat content in some Nigeria pepper is obtained using the proposed calibration product type estimator and the conventional product type estimator. This will help to compare the precision of the proposed estimators. The data set used is from Ojua et al. [10] for sensitivity of some micronutrient composition in two Nigerian peppers to treatment with different mutagens with two variables: Ash and Fat. Supposed an estimate of the mean fat content \bar{Y} in the pepper is of interest using ash as auxiliary variable and \bar{X} is assumed to be known. The data summary is presented:

$$N = 84, n = 43, \bar{X} = 5.002, \bar{Y} = 1.8042, L = 2 \quad \rho = -0.892 \quad R = 0.3607 \quad s_x^2 = 15.1722, \bar{Y} = 1.8042.$$

The results of the analysis using excel work sheet is presented in Tables.

Table 1 below shows the estimate for the mean fat in pepper, of the proposed calibration product type estimators with real-life data, under two constraints and the estimate for the conventional product type estimator. It was observed that the ratio type calibration estimator \bar{y}_{pcp42} obtained from the chi-square distance measure under two constraints gave a more precised estimate of the population mean than the other estimators. It was also observed that the estimator \bar{y}_{pcp5} gave the same estimate of the population mean as the conventional product type estimator.

Table 2 shows the variance estimates for the proposed calibration product type estimators and the conventional product type variance estimator. It was observed that the regression type calibration variance estimator $\hat{V}(\bar{y}_{pcp41})$ obtained from the chi-square distance measure under two constraints gave a minimum variance.

Table 1. Mean fat estimates for the proposed calibration product type estimators

Estimators	Estimates
\bar{y}_p	1.7271
\bar{y}_{pcp41}	1.5893
\bar{y}_{pcp42}	1.7274
\bar{y}_{pcp5}	1.7271
\bar{y}_{pcp61}	1.3860
\bar{y}_{pcp62}	1.4096

Table 2. Estimate of variance estimators for the proposed calibration product type estimator

Variance estimators	Estimates
$\hat{V}(\bar{y}_p)$	0.02579
$\hat{V}(\bar{y}_{pcp41})$	0.002538
$\hat{V}(\bar{y}_{pcp42})$	0.002539
$\hat{V}(\bar{y}_{pcp51})$	0.02579
$\hat{V}(\bar{y}_{pcp52})$	0.02579
$\hat{V}(\bar{y}_{pcp61})$	0.03150
$\hat{V}(\bar{y}_{pcp62})$	0.03680

4.2 Simulation study

To further examine the performance of the proposed calibration product type estimators for population mean, a simulation study was done for R = 10,000 runs using different sample sizes using R software with seed of (1113329), under Normal distribution, Gamma distribution and Exponential distribution.

Table 3, 4 and 5 show the percent average relative efficiency($\% \overline{RE}$), percent average absolute bias ($\% \overline{ARB}$), and average coefficient of variation (\overline{CV}) under two constraints for Normal distribution, Gamma distribution and Exponential distribution respectively using different sample sizes of 10%, 15%, 20% and 25%. It was observed from Table 3 that the proposed calibration product type estimators were more efficient compared to the conventional product time estimator. Also, under the distributional assumption of exponential distribution, the proposed estimators were most efficient as compared to when the distributional assumption is gamma and normal in nature. The highest efficiency was observed when the sample size was assumed to be 15%, however, the efficiency was not sample size dependent because, when the sample size was increased to 20% the efficiency dropped and still increased at 25%. Never the less, under exponential distribution relative efficiency was higher across sample sizes than gamma and normal distribution (Table 3).

From Table 4 the proposed estimators were shown to be more consistent as compared to the conventional product type estimator and also, when the distributional postulation was exponential in character the proposed estimators were seen to be more consistent, than when the distributional assumption is gamma and normal in character. Similarly, the increase in sample sizes did not make much of a difference in the consistency of the proposed calibration product type estimators. It was also observed that the proposed estimators are more reliable, under the gamma and exponential distribution, with exponential distribution taking the lead. Also under the normal distribution the conventional product type estimator and the proposed calibration product type estimators were not reliable. Also it was observed that as the sample size increases there was no significant increase in the reliability of the estimators (Table 5).

The above observations are all pointers to the fact that the proposed calibration product type estimators in this present study are more efficient, consistent and reliable estimators as compared to the conventional product type estimator.

Table 3. Percent average relative efficiency for gamma, normal and exponential distribution

Sample size	Distributions	\bar{y}_p	\bar{y}_{pcp41}	\bar{y}_{pcp42}	\bar{y}_{pcp61}	\bar{y}_{pcp62}
10%	GAMMA	100	400.45	726.13	1319.35	1319.35
	NORMAL	100	55.00	63.88	70.14	70.14
	EXPONENTIAL	100	70488.01	1103.17	723.17	723.17
15%	GAMMA	100	400.75	730.53	1331.05	1331.05
	NORMAL	100	54.91	63.82	70.06	70.06
	EXPONENTIAL	100	208068.87	1094.49	716.08	716.08
20%	GAMMA	100	401.28	734.28	1342.87	1342.87
	NORMAL	100	55.10	64.09	70.35	70.35
	EXPONENTIAL	100	65990.84	1090.96	716.47	716.47
25%	GAMMA	100	401.77	735.97	1349.51	1349.51
	NORMAL	100	55.01	63.99	70.26	70.26
	EXPONENTIAL	100	73407.10	1090.43	717.10	717.10

Table 4. Percentage average absolute relative bias for gamma, normal and exponential distribution

	Distributions	\bar{y}_p	\bar{y}_{pcp41}	\bar{y}_{pcp42}	\bar{y}_{pcp61}	\bar{y}_{pcp62}
10%	GAMMA	266.893	66.6	36.7	20.2	20.2
	NORMAL	47.623	86.6	74.5	67.8	67.8
	EXPONENTIAL	1018.364	1.4	92.3	140.8	140.8
15%	GAMMA	267.119	66.6	36.6	20.1	20.1
	NORMAL	47.538	86.6	74.5	67.8	67.8
	EXPONENTIAL	1018.813	1.5	93.1	141.7	141.7
20%	GAMMA	267.400	66.6	36.3	19.9	19.9
	NORMAL	47.709	86.6	74.4	67.8	67.8
	EXPONENTIAL	1016.788	1.5	93.2	141.9	141.9
25%	GAMMA	269.618	66.6	36.4	19.8	19.8
	NORMAL	47.628	86.5	74.4	67.7	67.7
	EXPONENTIAL	1016.097	1.4	93.2	141.7	141.7

Table 5. Average coefficient of variation for gamma, normal and exponential distribution

Sample size	Distributions	\bar{y}_p	\bar{y}_{pcp41}	\bar{y}_{pcp42}	\bar{y}_{pcp61}	\bar{y}_{pcp62}
10%	GAMMA	266.893	66.648	36.756	20.229	20.229
	NORMAL	47.623	86.582	74.547	67.899	67.889
	EXPONENTIAL	1018.364	1.445	92.312	140.819	140.819
15%	GAMMA	267.119	66.656	36.565	20.068	20.068
	NORMAL	47.538	86.581	54.493	67.855	67.855
	EXPONENTIAL	1018.813	1.548	93.086	141.683	141.683
20%	GAMMA	267.400	66.636	36.654	19.913	19.913
	NORMAL	47.709	86.580	74.445	67.820	67.820
	EXPONENTIAL	1016.788	1.541	93.201	141.917	141.917
25%	GAMMA	269.618	66.609	36.363	19.831	19.831
	NORMAL	47.628	86.575	74.428	67.783	67.783
	EXPONENTIAL	1016.097	1.384	93.183	141.694	141.694

5 Conclusion

In this paper, we proposed calibration product type estimators of population mean in stratified sampling to be used in survey when the variate are negatively correlated. The performance of the proposed estimators was compared using real – life and simulated data set. It was shown that the calibration product type estimators obtained by minimizing the chi-square distance measure gave a better estimator with minimum variance than the

other estimators obtained from the minimum entropy and modified chi-square distance measures. Also when the underlying distribution is exponential in nature, the proposed estimators outperform the conventional product type estimator.

Competing Interests

Authors have declared that no competing interests exist.

References

- [1] Robson DS. Application of multivariate polykays to the theory of unbiased ratio-type estimation. *Journal of the American Statistical Association*. 1957;2:511-522.
- [2] Murthy MN. Product Method of Estimation. *The Indian Journal of Statistics, Series A*. 1964;26(1):69-74.
- [3] Perri PF. Combining two auxiliary variables in ratio-cum-product type estimators. *Proceedings of Italian Statistical Society. International meeting on Statistics and Environment, Messina*. 2005;193-196.
- [4] Solanki RS, Singh HP, Rathour A. An alternative estimator for estimating the finite population mean using auxiliary information in sampling surveys. *International Scholarly Research Network, ISRN Probability and Statistics*. 2012;12.
- [5] Deville JC, Sarndal CE. Calibration estimators in survey sampling. *Journal of the American Statistical Association*. 1992;87(418):376-382.
- [6] Tracy DS, Singh S, Arnab R. Note on calibration in stratified and double sampling. *Survey Methodology*. 2003;29(1):99-104
- [7] Clement EP, Enang EI. On the efficiency of ratio estimators over regression estimators. *Communication in Statistics-Theory and Methods*. 2017;46(11):5357-5367.
- [8] Koyuncu N, Kadilar C. Calibration estimator using different distance measures in stratified random sampling. *International Journal of Modern Engineering Research*. 2014;3(1):415-419.
- [9] Singh S, Arnab R. On calibration of design weights. *Metron-International Journal of Statistics*. 2011;69(2):185-205.
- [10] Ojua E, Abu N, Omeke J. Data for sensitivity of some micronutrient composition in two Nigerian pepper to treatment with different mutagens. *Mendeley Data*. 2018;1.
Available:<http://dx.doi.org/10.17632/kyvd29yxcj.1>

© 2021 Ojua et al.; This is an Open Access article distributed under the terms of the Creative Commons Attribution License (<http://creativecommons.org/licenses/by/4.0>), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Peer-review history:

The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)

<https://www.sdiarticle4.com/review-history/65900>