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Synchronization in the Single-Manufacturer Multi-Buyer Integrated Inventory Supply Chain: Derived Without Derivatives

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Abstract

The significant advances in information and communication technologies and the growing focus on E-business and modern supply chain management have pushed firms towards closer collaboration and more efficient integration of the supply chain functions. In this paper, we consider two known integrated supply chain inventory models. In the first model the vendor ships a batch for each buyer immediately after the completion of batch production, whereas in the second model, the buyer receives the batch only after the previous batch is consumed by demand. We derived the closed form solutions for both models using a simple algebraic approach that can be easily applied without differential calculus. The proposed solution is illustrated by a numerical example.

Keywords: Supply chain management, inventory coordination, without derivatives, information sharing.

1 Introduction

The increasing use of internet and other related emerging information technologies is fast becoming vital tool for supply chain management and integration of the different supply chain functions. Research findings in this area revealed that information sharing and coordinated inventory replenishments can lead to significant reduction in the inventory and order costs as well as transportation costs. Real-time based exchange and sharing of critical information among the supply chain partners can increase the efficiency and effectiveness of the supply chain. For example, Gunasekaran and Ngai [1] reviewed the literature related to information systems in the supply chain management and integration and found that information technology can play critical role to foster supply chain competitiveness and has a great influence on the effectiveness of the supply chain management. Ketikidis et al. [2] investigated the current use of information systems to support logistics and supply chain management in South East Europe. They identified major challenges and developments on the use of information systems for logistics and supply chain

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management by enterprises. They examined the actual level of satisfaction of current policy on LSCM, and they revealed the actual need of enterprises in South East Europe on effective use of information systems for logistics and supply chain management. Lee et al. [3] in their investigation on the bullwhip effect in supply chains reported that lack of information sharing can lead to excessive inventory, poor customer service, lost revenues, unplanned capacities, and ineffective logistics. They recommend avoiding managerial independence by integrating various supply chain functions. Su and Yang [4] looked into the adoption of ERP systems and the impacts on firm competence in supply chain management. They proposed a model featuring ERP benefits to firm competences in supply chain management. The data collected from experts in Taiwanese IT firms provided empirical evidence that adoption of ERP can lead to better supply chain management competence at strategic planning level as well as at operational process, customer and relationship, and planning and control process integration levels.

In recent years, the issue of inventory coordination between a single-vendor and multi-buyers has received considerable interest in the supply chain management literature [5-6]. Kim et al. [7] developed an integrated analytical model for the synchronized procurement, production and delivery policies in the single vendor multi- retailers supply chain system. They proposed a heuristic solution algorithm and presented numerical examples to illustrate the application of the proposed algorithm. Chan and Kingsman [8] developed a mathematical model for the single-vendor multi-buyers supply chain inventory coordination problem assuming synchronized delivery and production cycles. Results from their numerical analysis have shown that the synchronized inventory decisions will achieve a significant reduction in the supply chain system costs.

Zavanella and Zanoni [5] proposed a synchronized single-vendor multi-buyer production model with shared management of the buyers' inventory according to the Consignment Stock policy. Hoque [9] developed a generalized synchronized single-vendor multi-buyer supply chain model in which the vendors' lot is transferred in equal and/or unequal sized batches. Through comparative studies with techniques developed by Zavanella and Zanoni [5], he reported significant cost reductions by his model. Hariga et al. [6] commented on Hoque [9] and showed that the cost comparison between his model and of the models by Hoque (2011) is not appropriate. They further demonstrated that the model in [9] resulted in a larger total cost per unit time if adjusted to fit to an appropriate comparative analysis with the model in [5].

Traditionally, differential calculus was the most popular approach for optimizing integrated production inventory supply chain models, Teng et al. [10]. However, another line of research focused on easy alternative non-differential calculus solution methods for the optimization of these types of systems. For example, Grubbström [11] introduced the use of algebraic optimization approach to the EOQ model with no backorders. Since then, algebraic approach for the optimization of production inventory models has received notable attention of researchers. The popularity of the algebraic approach for the optimization of production inventory models could be due to the fact that it requires basic knowledge of simple elementary mathematics, Cárdenas-Barrón [12]. Examples on the use of algebraic procedures to solve the inventory models include: Cárdenas-Barrón [13], Cárdenas-Barrón [14], Chung and Wee [15], Wee and Chung [16], Chi [17], Seliaman and Ahmad [18], Ben-Daya et al. [19] and Cárdenas-Barrón [20]. Cárdenas-Barrón [14] reviewed the algebraic procedure used by different researchers between (1995) and (2006) to solve the inventory problems. Another intensive and relatively recent review on the use of algebraic optimization methods in the development of production inventory systems is presented in Cárdenas-Barrón [12].

In this paper, we revisit the models in Hoque [21] and propose a simpler closed form solution for the single-manufacturer multi-buyer integrated supply chain inventory coordination problem. The remainder of this paper is organized as follows. The next section presents the problem definition, assumptions, notations, and the detailed development of the models. A numerical example is presented in section 3. Section 4 contains some concluding remarks.

2 Models Development

2.1 Notations and Assumptions

As in Hoque [21], the following notations are used in developing the model:

D =The total demand rate in the supply chain per year;
P_i=Production rate at the vendor;
h_i =Inventory holding cost per item per year for the vendor;
S=Vendor's production set up cost per lot;
Z=the size of batch transferred from the vendor to the buyers;
n= the number of equal sized batches in a lot;
D_i=The annual demand rate the i^{th} buyer;
h_i =Inventory holding cost per item per year for the i^{th} buyer;
S_i =Order cost the i^{th} buyer per order.

Assumptions for the single-vendor multi-buyers supply chain model:

(a) A single product is produced by the vendor and transferred to the buyers;
(b) Replenishment is instantaneous;
(c) Production rates and Demand rate are deterministic and uniform;
(d) A lot produced by the vendor and sent in equal batches to the buyers;
(e) Complete information sharing policy is adopted;
(f) There is no backlogging.

2.2 The First Model

In this model the vendor ships a batch for each buyer immediately after the completion of batch production. Therefore, the total cost for the vendor consists of the inventory carrying cost and production set-up cost. In each production cycle, the vendor produces a lot of size *nZ*. Each batch of size *Z* is transferred to the buyers as soon as its processing is finished, so the total cost for the vendor per year is:

$$TC_v = \frac{DhZ}{2P} + \frac{DS}{nZ} \tag{1}$$

Hoque [21] has shown that the average annual total cost of inventory, transportation and ordering for the ith buyer is given by:

$$TC_{Bi} = \frac{D_i Z h_i}{2D} + \frac{(n-1)}{2}\left(\frac{1}{D} - \frac{1}{P}\right) D_i h_i Z + D(S_i + nT_i)/nZ \tag{2}$$

Hence the total annual cost for the integrated vendor-buyers supply chain is:

$$C = \frac{Z}{2}\left[\frac{Dh}{P} + \left\{\frac{1}{D} + (n-1)\left(\frac{1}{D} - \frac{1}{P}\right)\right\}\sum_{i=1}^{m} D_i h_i\right] + \frac{1}{Z}\left[D\left\{\frac{S + \sum_{i=1}^{m} S_i}{n} + \sum_{i=1}^{m} T_i\right\}\right] \tag{3}$$

Equation (3) can be rewritten as:

$$TC = ZY + \frac{W}{Z} \tag{4}$$

Where

$$Y = \frac{\left[\frac{Dh}{P} + \left\{\frac{1}{D} + (n-1)\left(\frac{1}{D} - \frac{1}{P}\right)\right\}\sum_{i=1}^{m} D_i h_i\right]}{2} \tag{5}$$

and

$$W = \left[D\left\{\frac{S + \sum_{i=1}^{m} S_i}{n} + \sum_{i=1}^{m} T_i\right\}\right] \tag{6}$$

Using the algebraic approach proposed by Cárdenas-Barrón [14], the annual total cost for the entire supply chain in Eq. (4) can be represented by factorizing the term 1/Z and completing the perfect square, one has:

$$TC = \frac{1}{Z}\left(Z^2 Y - 2Z\sqrt{YW} + W + 2Z\sqrt{YW}\right)^2 \tag{7}$$

Factorizing the perfect squared trinomial in a squared binomial we obtain:

$$TC = \frac{1}{Z}\left(Z\sqrt{Y} - \sqrt{W}\right)^{2^2} + 2\sqrt{YW} \tag{8}$$

Hence, the optimal batch size Z^* is:

$$Z^* = \sqrt{\frac{W}{Y}} \tag{9}$$

Now, the minimum annual total cost for the entire supply chain is:

$$TC = 2\sqrt{YW} \tag{10}$$

The optimal batch size Z^* is a function of the integer multiplier n. We use the method of perfect square to drive the optimal value of this integer multiplier. Substituting for Y and W into Eq.(10) we get:

$$TC = \sqrt{2}\left\{(na+b)\left(c+\frac{d}{n}\right)\right\}^{\frac{1}{2}} \tag{11}$$

Where:

$$a = \left(\frac{1}{D}-\frac{1}{P}\right)\sum_{i=1}^{m} D_i h_i\ ,\ b = \frac{Dh}{P}+\frac{1}{P}\sum_{i=1}^{m} D_i h_i\ ,\ c = D\sum_{i=1}^{m} T_i \ \text{ and } d = D\left(S+\sum_{i=1}^{m} S_i\right)$$

Rewriting Eq.(11):

$$\sqrt{2}\left\{\frac{1}{n}\left[n\sqrt{ac}-\sqrt{bd}\right]^2 + \left[\sqrt{ad}+\sqrt{bc}\right]^2\right\}^{\frac{1}{2}} \tag{12}$$

From (12) setting:

$$\left[n\sqrt{ac}-\sqrt{bd}\right]^2 = 0,$$

the optimal value of the integer multiplier n is derived as follows:

$$n^* = \sqrt{\frac{bd}{ac}} \tag{13}$$

Chung and Wee [15] pointed that since the value n* is a positive integer, the following condition must be satisfied:

$$(n^*).(n^*-1) \le \left(\frac{bd}{ac}\right) \le (n^*).(n^*+1) \tag{14}$$

Following the procedure described by García-Laguna et al. [22], the optimal integer multiplier must have one of the following two values:

$$n_1^* = \left\lceil -0.5+\sqrt{0.25+bd/ac}\ \right\rceil \text{ or } n_2^* = \left\lfloor 0.5+\sqrt{0.25+bd/ac}\ \right\rfloor \tag{15}$$

Where $\lceil x \rceil$ is smallest integer greater than or equal to x and $\lfloor x \rfloor$ is largest integer less than or equal to x.

If $n_1^* = n_2^*$, then there is a unique optimal integer value and we can substitute n* from Eq.(15) into Eq.(9) to find the optimal batch size Z^*. Also, substituting n^* from Eq.(15) into Eq.(12) derives the optimal annual total cost in the following closed form:

$$TC^* = \sqrt{2}\left[\sqrt{ad} + \sqrt{bc}\right] \tag{16}$$

If we substitute for a, b, c and d into Eq.(15), we get:

$$TC^* = \sqrt{2}\left[\sqrt{D\left(\frac{1}{D} - \frac{1}{P}\right)\left(S + \sum_{i=1}^{m} S_i\right)\sum_{i=1}^{m} D_i h_i} + \sqrt{D\left(\frac{Dh}{P} + \frac{1}{P}\sum_{i=1}^{m} D_i h_i\right)\sum_{i=1}^{m} T_i}\right] \tag{17}$$

Eq.(17) represents the minimum annual total cost for the integrated supply chain system expressed only in terms of the production rate, demand rate and cost parameters. If $n_1^* \neq n_2^*$, then there are two optimal integer values for the multiplier *n* as shown by García-Laguna et al. [22].

2.3 The Second Model

In his second model, Hoque [21] assumed that the vendor transfers a batch of size Z to the buyers after every *Z/D* units of time. For this case, he formulated the total annual cost for the integrated vendor-buyers supply chain as:

$$TC = \frac{Z}{2}\left[\frac{Dh}{P} + \frac{\sum_{i=1}^{m} D_i h_i}{D} + (n-1)\left(\frac{1}{D} - \frac{1}{P}\right)Dh\right] + \frac{1}{Z}\left[D\left\{\frac{S + \sum_{i=1}^{m} S_i}{n} + \sum_{i=1}^{m} T_i\right\}\right] \tag{18}$$

Eq. (18) can be rewritten as:

$$TC = ZY + \frac{W}{Z} \tag{19}$$

Where:

$$Y = \frac{\left[\frac{Dh}{P} + \frac{\sum_{i=1}^{m} D_i h_i}{D} + (n-1)\left(\frac{1}{D} - \frac{1}{P}\right)Dh\right]}{2} \tag{20}$$

and

$$W = \left[D \left\{ \frac{S + \sum_{i=1}^{m} S_i}{n} + \sum_{i=1}^{m} T_i \right\} \right] \tag{21}$$

To drive the optimal values of Z and n, we can follow the same algebraic approach detailed in the previous subsection. Keeping the definitions of c and d as used in manipulation of the first model, and defining:

$$e = \left(\frac{1}{D} - \frac{1}{P} \right) Dh \text{ and } f = \frac{2Dh}{P} + \frac{1}{D} \sum_{i=1}^{m} D_i h_i - h$$

The optimal batch size *Z* is:

$$Z^* = \sqrt{\frac{W}{Y}} \tag{22}$$

and the optimal integer *n* is

$$n^* = \sqrt{\frac{fd}{ec}} \tag{23}$$

Similar to the step in the first model, the value of n* is a positive integer, the following condition must be satisfied:

$$(n^*).(n^* - 1) \le (\frac{fd}{ec}) \le n^*).(n^* + 1) \tag{24}$$

By substituting for Z^*, n^* in Eq.(18), the optimal annual total cost is given in the following closed form in case of unique optimal integer value for *n*:

$$TC^* = \sqrt{2} \left[\sqrt{ed} + \sqrt{fc} \right] \tag{25}$$

If we substitute for e, f, c and d into Eq. (23), we get:

$$TC^* = \sqrt{2} \left[D \sqrt{h \left(\frac{1}{D} - \frac{1}{P} \right) \left(S + \sum_{i=1}^{m} S_i \right)} + \sqrt{\left(\frac{2Dh}{P} + \frac{1}{D} \sum_{i=1}^{m} D_i h_i - h \right) D \sum_{i=1}^{m} T_i} \right] \tag{26}$$

3 Numerical Examples

Consider the single-manufacturer multi-buyer supply chain system used by reference [21] with the buyers' data given in Table 1, and the data for the manufacturer is as follows:
S=300, *h*=0.2, *P*=1500, and *D*=970.

Table 1. Data for the 5 buyers

Buyer$_i$	s_i	D_i	h_i	T_i
1	25	200	0.22	25
2	15	150	0.24	20
3	25	225	0.25	18
4	30	230	0.23	25
5	30	165	0.21	15

Applying our alterative algebraic procedure on this example procedure provides an optimal lot size of 3277.6 for the manufacturer. The optimal number of shipments is given as 4. The total cost under this solution is 495.4. The details of the cost components resulting from this solution are given in Table 2. This solution is identical to the solution presented in [21].

Table 2. The solution of the numerical example

Cost component	**Our solution**	**Solution in [21]**
Lot size, batch sizes and its no.	3277.6	3277.6
Set up cost of the manufacturer	88.8	88.8
Inventory cost of the manufacturer	53.0	53.0
Ordering costs of buyers	37.0	37.0
Inventory costs of the buyers	194.7	194.7
Transportation cost	121.9	121.9
Total cost of the manufacturer	141.8	141.8
Total cost of the buyers	353.6	353.6
Total cost	495.4	495.4

Many models were developed in the literature for motivating the buyers to accept the centralized inventory coordination between the buyers as a group and the vendor. The buyers can get benefits of sharing their information and consequently synchronize their inventory replenishment cycle to reduce their transportation and set-up cost. This is especially possible if the buyers are located in the same region (Sarmah et al. [23], Seliaman [24]. Coordinating their transportation management, the buyers can significantly reduce the total costs and improve their service levels (Chan and Zhang [25], Ertogral et al. [26], Çapar et al. [27], Kang and Kim [28]. The buyers can also jointly reduce their ordering costs (Woo et al. [29]).

4 Conclusion

This paper addressed two integrated inventory models for coordinating the inventory policies between a vendor and a group of buyers. In the first model the vendor ships a batch for each buyer immediately after the completion of batch production, whereas in the second model, the buyer receives the batch only after the previous batch is consumed by demand. The study proposed a simpler algebraic method to solve the two models without using differential calculus. We derived the closed-form solutions for both models. The described models can be used by the buyers to get benefits of vertically sharing their information and synchronizing their inventory replenishment cycle. The group of buyers can use either Eq.(17) or Eq.(26) to negotiate the manufacturer to offer them a discounted price based on their horizontal synchronization of the inventory replenishment cycle. The vendor should entice the buyers to adopt the coordination since his total cost will be minimized under the coordination. After compensating the buyers for their increased costs as a group, savings resulting from the coordination should be fairly shared between the vendor and the group of buyers. In turns, the buyers as a group should compensate any individual buyer for any cost difference between his/her cost under the synchronized vertical cycle in his/her optimal independent cost.

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Competing Interests

Author has declared that no competing interests exist.

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