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Statistical Properties of Buys-Ballot Estimates for Multiplicative Model with the Error Terms

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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ABSTRACT

In this study, we discuss statistical properties of Buys-Ballot estimates for multiplicative model with their error terms. The aim of this study is to characterize the properties of the row, column and overall means and variances of the Buys-Ballot table for multiplicative model with the error terms. The properties of Buys-Ballot estimates in this study are used for (1) estimation of trend parameters (2) estimation of seasonal effect (3) choice of model for decomposition. The results indicate that (1) the column variance ($\hat{\sigma}_j^2$) of the Buys-Ballot depends on the seasonal indices (S_j^2) of the j^{th} seasons. (2) the model that best describe the pattern in the transformed series is additive. This further confirms that the appropriate model of the original series is multiplicative.

Keywords: *Time series decomposition; multiplicative model; error term; trend parameters; seasonal indices; choice of model; buys-ballot table.*

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1. INTRODUCTION

Dozie [1] provided an estimation method based on the row, column and overall means and variances of the Buys-Ballot table for the mixed model in descriptive time series. This method was initially developed for short period of time in which the trend-cycle component (M_t) is jointly combined and can be represented by linear equation.

$M_t = a + b_t$, t = 1, 2, ..n, where a is the intercept, b is the slope and t is the time point.

The models most commonly used for time series decomposition are the

Additive Model:

$$X_t = T_t + S_t + C_t + e_t \qquad (1)$$

Multiplicative Model:

$$X_t = T_t \times S_t \times C_t \times e_t \qquad (2)$$

and Mixed Model

$$X_t = T_t \times S_t \times C_t + e_t \qquad (3)$$

If short period of time are involved, the cyclical component is superimposed into the trend [2] and the observed time series $(X_t, t = 1, 2, ..., n)$ can be decomposed into the trend-cycle component (M_t) , seasonal component (S_t) and the irregular/residual component (e_t) Therefore, the decomposition models are

Additive Model:

$$X_t = M_t + S_t + e_t \qquad (4)$$

Multiplicative Model:

$$X_t = M_t \times S_t \times e_t \qquad (5)$$

and Mixed Model

It is always assumed that the seasonal effect, when it exists, has period *s*, that is, it repeats after *s* time periods.

$$S_{t+s} = S_t, \text{ for all } t \qquad (6)$$

For Equation (4), it is necessary to make the further assumption that the sum of the seasonal components over a complete period is zero, ie ,

$$\sum_{j=1}^{s} S_{t+j} = 0 . \qquad (7)$$

Similarly, for Equations (5) and (6), the convenient variant assumption is that the sum of the seasonal components over a complete period is *s*.

$$\sum_{j=1}^{s} S_{t+j} = s .$$

Dozie and Nwanya [3] provided the expected values of parameters of trend and seasonal indices for both multiplicative and mixed models with their error terms.

Iwueze and Nwogu [4] stated that, the seasonal variances of the Buys-Ballot table are constant for the additive model, but contains the seasonal effect for the multiplicative model. In addition, in obtaining, the row, column and overall averages and variances of additive and multiplicative models Iwueze and Nwogu [4] did not consider the error term.

Iwueze, *et al,* [5] proposed uses of the Buys-Ballot table in time series which include (1) choice of model, (2) choice of transformation (3) estimation of trend parameters and seasonal effects. They provided the use of the relationship between the seasonal means $(\overline{X}_{.j}, j = 1, 2, ..., s)$ and the seasonal standard deviations $(\hat{\sigma}_{.j}, j = 1, 2, ..., s)$ to choose the appropriate model for decomposition. The model structure is additive, if the seasonal standard deviations indicate no appreciable increase or decrease relative to any increase or decrease in the seasonal means. On the other hand, a multiplicative model is usually appropriate when the seasonal standard deviations show appreciable increase/decrease relative to any increase /decrease in the seasonal means. Oladugba, et al, [6] proposed decomposition models between additive and multiplicative. They stated that, the seasonal fluctuation exhibits constant amplitude with respect to the trend in additive case while amplitude of the seasonal fluctuation is a function of the trend in multiplicative seasonality.

2. METHODOLOGY

Basic properties of Buys-Ballot estimates for multiplicative model with the error terms in this study are done using Buys-Ballot method usually referred to in the literature. This method adopted in this study assumed that the data are arranged in a Buys-Ballot table with m rows and s columns. For details of this method see Wei [7], Iwueze et al [5], Nwogu et al [8], Dozie and Ihekuna [9], Dozie et al [10], Dozie and Ijeomah [11], Dozie and Ibebuogu [12], Dozie and Uwaezuoke [13], Dozie and Ihekuna [14], Dozie and Ibebuogu [15]

For the multiplicative model, the row, column and overall averages and variances obtained with the error terms are given in equations (8) to (10). From equation (12) it is clear that the column variances ($\hat{\sigma}_j^2$) depends on the seasons j only through the square of the seasonal indices (S_j^2) and the trending curves through the square of seasonal average with the error variance.

2.1 Buys-Ballot Estimates of Row, Column and Overall Means with the Error Terms

The summaries of the row, column and overall means and variances with the error terms are given in equations (8) to (10) for linear trending curve under the multiplicative model.

$$\overline{X}_{i.} = \left[a - bs + \frac{b}{s}\sum_{j=1}^{s} jS_j + bsi \right] * \overline{e}_{i.} \quad (8)$$

$$\overline{X}_{.j} = \left[a\,\overline{e}_{.j} + \frac{bs}{m}\sum_{i=1}^{m} i\,e_{ij} - bs\,\overline{e}_{,j} + bj\,\overline{e}_{.j} \right] * S_j \quad (9)$$

$$\overline{X}_{..} = a + b\left(\frac{n-s}{2}\right) + bC_1 \quad (10)$$

2.2 Properties of Row, Column and Overall Means of with the Error Terms

(1) It is a function of trending curve with error term
(2) A function of trending curve and seasonal effect with error term
(3) A product of both row and column specific with error terms

2.3 Estimates of Periodic, Seasonal and Overall Variances with the Error Variances

The summaries of the periodic, seasonal and overall variances with their error variances are given in equations (11) to (13) for linear trending curve under the multiplicative model.

$$\hat{\sigma}_{i.}^2 = \left\{ \left[(a + bs(i-1)) + bC_1\right]^2 + \mathrm{var}\begin{bmatrix} [a + bs(i-1)]S_j \\ + bjS_j \end{bmatrix} \right\} \sigma_2^2 \quad (11)$$

$$\hat{\sigma}_{.j.}^2 = \left\{ \frac{b^2(n^2 - s^2)}{12} + \left[a + b\left(\frac{n-s}{2}\right) + bj \right]^2 \right\} S_j^2\ \sigma_2^2 \quad (12)$$

$$\hat{\sigma}_x^2 = \left\{ \begin{array}{l} \frac{b^2(n^2 - s^2)}{12} + \left[a + b\left(\frac{n-s}{2}\right) + C_1 \right]^2 \\ + \left[a^2 + 2ab\left(\frac{n-s}{2}\right) + \frac{b^2(n-s)(2n-s)}{6} \right] \mathrm{Var}(S_j) \\ + b^2\,\mathrm{Var}(jS_j) + 2b\left[a + b\left(\frac{n-s}{2}\right) \right] \mathrm{Cov}(S_j, jS_j) \end{array} \right\} \sigma_2^2 \quad (13)$$

2.4 Properties of Row, Column and Overall Variances with the Error Variances

(1) It is a product of quadratic function of the season (j) and square of the seasonal effect with error variance.
(2) A function of season (j) through the seasonal effect s_j^2 with error variance
(3) A product of sum of squares and cross-products trend parameters and seasonal effect with error variance

(4) A product of variance of S_j and co-variance of jS_j

These properties are what could be used for (1) estimation of trend parameters (2) estimation of seasonal effect and (3) choice of appropriate model for decomposition.

The Buys-Ballot estimates of trend parameters and seasonal indices are provided in Table 1. Table 2 contains the Buys-Ballot estimates of trend parameters and seasonal indices when there is no trend (when b = 0).

Table 1. Estimates of trend parameters and seasonal indices for multiplicative model

Parameter	Multiplicative Model
a	$a+\hat{b}(s-c_1)$
b	$\frac{\beta}{s}$
S_j	$\frac{\bar{X}_{.j}}{a+b\left(\frac{n-s}{2}\right)+bj}$

Table 2. Estimate of trend parameters and seasonal indices when there is no trend

Estimate	Multiplicative Model
$\bar{X}_{i.}$	a
$\bar{X}_{.j}$	a
$\bar{X}_{..}$	a
S_j	$\frac{\bar{X}_{.j}}{a}$

2.5 Levene's Test for Constant Variance

The Levene's test statistic for the null hypothesis

$H_0:\sigma_i^2=\sigma_j^2$

$H_1:\sigma_i^2\neq\sigma_j^2$ *for at least one* $i\neq j$ is defined as

$$W=\frac{(N-K)\sum_{i=1}^{k}N_i\left(\bar{z}_{i}-\bar{z}_{..}\right)^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{N_i}\left(z_{ij}-\bar{z}_i\right)^2} \quad (14)$$

where k is the number of different groups, N_i *is the number of cases* in the *ith group*, Y_{ij} *is the value of the jth* observation in the *ith group*.

z_{ij} may be defined as deviation of y_{ij} from the mean $(\bar{y}_i)$ or from the median $(\bar{y}_i)$. That is

$$z_{ij}=y_{ij}-\bar{y}_i \;\; or \;\; y_{ij}-\bar{y}_i \quad (15)$$

$\bar{z}_{i.}=\frac{1}{N_i}\sum_{j=1}^{N_i}z_{ij}$ is the mean of the z_{ij} for group i (16)

$\bar{z}_{..}=\frac{1}{N}\sum_{i=1}^{k}\sum_{j=1}^{N_i}z_{ij}$ is mean of all z_{ij}. (17)

The test statistic W approximately follows the F-distribution with $k-1$ *and* $N-K$ degree of freedom. To suit the Buys-Ballot procedure, the levene's test statistic is modified with

$N=ms,\; k=s\; N_i=m$ as

$$W=\frac{(ms-s)}{s-1}\left[\frac{\sum_{j=1}^{s}m(\bar{z}_{i}-\bar{z}_{..})^2}{\sum_{j=1}^{s}\sum_{i=1}^{m}\left(z_{ij}-\bar{z}_{.j}\right)^2}\right] \quad (18)$$

$$=\frac{s(m-1)}{s-1}\left[\frac{m\sum_{i=1}^{s}\left(\bar{z}_{i}-\bar{z}_{..}\right)^2}{\sum_{i=1}^{s}\sum_{j=1}^{m}\left(z_{ij}-\bar{z}_{.j}\right)^2}\right] \quad (19)$$

2.6 Chi-Square Test

The seasonal variance of the Buys-Ballot table for the mixed model with error variance $\sigma_{zj}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2$ is reduces to that of test null hypothsis.

$$H_0: \ \sigma_j^2 = \sigma_{zj}^2$$

and the appropriate model is mixed

$$H_1: \ \sigma_j^2 \neq \sigma_{zj}^2$$

and the appropriate model is not mixed

$\sigma_j^2 = (j = 1, \ 2, ... s)$ *is the true variance of the jth season.*

$$\sigma_{zj}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2 \qquad (20)$$

and

σ_1^2 *is the error variance assumed to be equal to* 1

Therefore, the statistic is $\chi_c^2 = \frac{(m-1)\sigma_j^2}{\sigma_{zj}^2}$ (21)

follows the chi-square distribution with $m-1$ degree of freedom, m is the number of observations in each column and s is the seasonal lag.

The interval $\left[\chi^2_{\frac{\alpha}{2},(m-1)}, \chi^2_{1-\frac{\alpha}{2},(m-1)}\right]$ *contains the statistic* (21) with $100(1-\alpha)$% degree of confidence.

2.7 Choice of Appropriate Transformation

For time data arranged in Buys-Ballot table Akpanta and Iwueze [16] provided the slope of the regression equation of log of group standard deviation on log of group mean as given in equation (22) is what is needed for choice of appropriate transformation. Some of the values of slope β and their implied transformation are stated in Table 3.

$$\log_e \hat{\sigma}_i = a + \beta \log_e \hat{X}_{i.} \qquad (22)$$

The method of Akpanta and Iwueze [16] is used in choosing the appropriate transformation, the natural logarithm of standard deviation will be used to regress against the natural logarithm of periodic means and the result of the β - value will determine the type of transformation.

2.8 Estimation of Trend Parameters and Seasonal Indices

The summaries of the Buys-Ballot estimates of trend parameters and seasonal indices for multiplicative model are given in Table 1. They are:

$$\bar{X}_{i.} = a - b(s - c_1) + (bs)i \qquad (23)$$

$$\equiv \alpha + \beta_i \qquad (24)$$

$$\hat{a} = \alpha + \hat{b}(s - c_1) \qquad (25)$$

$$\hat{b} = \frac{\beta}{s} \qquad (26)$$

$$S_j = \frac{\bar{X}_j}{a + b\left(\frac{n-s}{2}\right) + b_j} \qquad (27)$$

3. EMPIRICAL EXAMPLE

In this section, we present empirical example to illustrate: (1) estimation of trend parameters (2) estimation of seasonal effect and (3) choice of model for decomposition. Results from estimates of trend parameters and seasonal indices are contained in Section 3.1. Section 3.2 present the choice of appropriate model. The time series plots of actual and transformed data sets are given in figure 1 and 2.

3.1 Results from Estimates of Trend Parameters and Seasonal Indices

The expression of linear trend and seasonal indices for multiplicative models given as

$$\bar{X}_{.j} = 2.584 + 0.0201j \qquad (28)$$

Using (25),(26) and (27)

$$\hat{b} = 0.0201$$

$$\hat{a} = 2.584 - 0.0201\left(\frac{120-12}{2}\right)$$

$$\hat{a} = 1.4986$$

$$\hat{S}_j = \frac{\bar{X}_{.j}}{2.584 + 0.0201_j}$$

Multiplicative model satisfies

$\left(\sum_{j=1}^{s} S_j = s\right)$ as in equation (5)

,

3.2 Choice of Model

The test statistic shown in equation (19) is used to choose the appropriate model for decomposition for the study series. The null hypothesis that the data is not additive is reject if W is greater than the tabulated value, for which $\alpha = 0.05$ level of significance and $m-1=11$ degree of freedom equal to 1.81 or do not reject H_0 otherwise. The critical (1.81) is greater than W. This is an indication that the model structure is not additive.

Table 3. Bartlett's transformation for some values of β

S/No	1	2	3	4	5	6	7
β	**0**	$\frac{1}{2}$	**1**	$\frac{3}{2}$	**2**	**3**	**-1**
Transformation	**No transformation**	$\sqrt{X_t}$	$\log_e X_i$	$\frac{1}{\sqrt{X_t}}$	$\frac{1}{X_t}$	$\frac{1}{X_t^2}$	X_t^2

Table 4. Estimates of trend parameters

Parameter	Multiplicative model values
a	1.4986
b	0.0201

Table 5. Estimates of seasonal indices

j	$\bar{X}_j$	$\hat{S}_j$
1	2.2380	0.8594
2	2.8480	1.0853
3	2.6060	0.9855
4	2.7018	1.0140
5	2.8670	1.0680
6	2.7110	1.0024
7	2.7860	1.0225
8	2.9392	1.0708
9	2.7190	0.9834
10	2.5305	0.9086
11	2.8780	1.0263
12	2.7500	0.9734
$\sum_{j=1}^{s} \hat{S}_j$		12.0000

Table 6. Deviations of the Observed Values from Averages ($Z_{ij} = \left| y_{ij} - \bar{y}_{,j} \right|$)

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{z}_{i.}$	$\sigma_{i.}$
2008	6.43	4.81	8.16	13.09	2.18	3.75	3.66	3.57	0.51	10.65	5.24	12.67	74.67	6.22	4.09
2009	6.40	1.76	1.83	2.08	5.17	0.75	0.67	3.58	0.50	2.33	6.75	6.67	38.50	3.21	2.42
2010	5.57	1.76	0.83	2.92	0.83	0.75	2.33	6.42	4.50	4.33	1.75	4.67	36.67	3.06	1.98
2011	3.41	0.26	4.83	8.08	3.83	1.25	1.33	4.42	3.50	5.33	2.25	2.33	40.83	3.40	2.14
2012	5.40	1.26	0.17	12.92	0.83	5.25	2.67	1.58	25.50	9.33	9.25	20.33	94.50	7.88	8.13
2013	2.57	0.76	1.17	16.92	5.17	6.25	0.33	12.58	5.50	16.67	16.25	3.33	87.50	7.29	6.50
2014	3.59	5.26	0.83	2.92	10.17	2.25	0.33	0.42	0.50	5.33	11.25	8.33	51.17	4.26	3.87
2015	7.51	5.26	1.17	4.92	3.17	2.75	0.33	1.58	2.50	1.33	5.75	5.33	41.67	3.47	2.25
2016	1.59	1.26	3.17	10.08	1.83	3.25	0.67	0.58	3.50	10.67	0.25	7.67	44.50	3.71	3.70
2017	1.59	2.27	0.17	3.08	7.83	4.75	3.33	2.42	5.50	13.33	10.75	0.33	55.33	4.61	4.12
2018	3.57	4.78	0.83	2.92	5.83	2.75	1.67	5.42	4.50	9.33	9.75	2.33	53.67	4.47	2.80
2019	4.41	1.78	4.83	7.08	4.83	2.75	1.33	4.42	4.50	12.67	9.75	10.67	69.00	5.75	3.58
Total	52.16	31.00	28.00	87.00	51.67	36.50	18.67	47.00	61.00	101.33	89.00	84.67	688.00		
$\bar{z}_{.j}$	4.35	2.58	2.33	7.25	4.31	3.04	1.56	3.92	5.08	8.44	7.42	7.06		4.78	
$\sigma_{.j}$	1.96	1.86	2.46	4.99	2.82	1.74	1.19	3.31	6.69	4.72	4.63	5.52			4.32

Table 7. Square of deviations of the observed values from seasonal averages

$\left(z_{ij}-\bar{z}_{.j}\right)^2$													
	Jan.	**Feb.**	**Mar.**	**Apr.**	**May**	**Jun.**	**Jul.**	**Aug.**	**Sept.**	**Oct.**	**Nov.**	**Dec.**	**total**
2008	4.3	4.7	34	34	4.6	0.5	4.5	0.1	21	4.9	4.7	32	149
2009	4.3	0.7	0.3	27	0.7	5.3	0.8	0.1	21	37.4	0.4	0.2	97.7
2010	1.5	0.7	2.3	19	12	5.3	0.6	6.3	0.3	16.9	32.1	5.7	102.5
2011	0.9	5.4	6.3	0.7	0.2	3.2	0.1	0.3	2.5	9.7	26.7	22.3	78.2
2012	1.1	1.8	4.7	32	12	4.9	1.2	5.4	416	0.8	3.4	176	660.6
2013	3.1	3.36	1.4	93	0.7	10	1.5	75	0.2	67.6	78.0	13.9	348.6
2014	0.6	7.11	2.3	19	34	0.6	1.5	12	21	9.7	14.7	1.6	124.5
2015	11	7.11	1.4	5.4	1.3	0.1	1.5	5.4	6.7	50.6	2.8	3.0	95.7
2016	7.6	1.78	0.6	8.0	6.1	0.1	0.8	11	2.5	4.9	51.4	0.4	95.4
2017	7.6	0.11	4.7	18	13	2.9	3.2	2.3	0.2	23.9	11.1	45.2	130.9
2018	0.6	4.69	2.3	19	2.3	0.1	0.1	2.3	0.34	0.8	5.4	22.3	59.9
2019	0.1	0.69	6.3	0.1	0.3	0.1	0.1	0.3	0.34	17.8	5.4	13.0	44.3
Total	42	38.17	66	274	87	33	16	121	493	244	236	335	1987

Table 8. Calculation of $m\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$ m = 13

$\bar{z}_{.j}$	$\bar{z}_{..}$	$\bar{z}_{.j}-\bar{z}_{..}$	$\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$	$12\times\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$
4.40	4.78	-0.44	0.18	2.23
2.57	4.78	-2.21	4.84	58.08
2.32	4.78	-2.45	6.00	72.03
7.26	4.78	2.47	6.10	73.21
4.32	4.78	-0.47	0.22	2.65
3.03	4.78	-1.74	3.03	36.33
1.57	4.78	-3.22	10.37	124.42
3.93	4.78	-0.86	0.74	8.88
5.08	4.78	0.30	0.09	1.08
8.44	4.78	3.66	13.40	160.75
7.42	4.78	2.64	6.97	83.64
7.06	4.78	2.28	5.20	62.38
				685.7

From appendix A and Table 8

$$W=\frac{12*(12-1)(685.7)}{(12-1)(1987.1)} = \frac{90512.4}{21857.6} = 4.14$$

Having stated that the data is not additive model, we have to choose between multiplicative and mixed models. The null hypothesis that the data admits the mixed model is rejected, if the statistic defined in equation (21) lies outside the interval $\left[\chi^2_{\frac{\alpha}{2},(m-1)},\chi^2_{1-\frac{\alpha}{2},(m-1)}\right]$ which for $\alpha=0.05$ level of significance and $m-1=11$ degrees of freedom, equals (3.8, 21.9) or do not reject H_0 otherwise, and from equation (21) the calculated values, χ^2_{cal} given in Table 9 are obtained. With the critical values (3.8 and 21.9), all the calculated values are outside the range, showing that the model structure is not mixed.

However, there is indication the choice of appropriate model may be affected by violation of the underlying assumptions, therefore, it is required to evaluate data for transformation to meet the constant variance and normality assumptions in the distribution. When the seasonal variances of the transformed data given in Table 12 are tested for constant variance, the computed test statistic from equation (19) is 0.78 and that of the critical value is 1.8, at $\alpha = 0.05$ level of significance and $m-1=11$ degrees of freedom. This shows that the variance is constant and the transformed data is additive model.

From appendix B and Table 9, $\sigma_1^2 = 1,\ b = 0.1143,\ n = 144,\ m = 12$

Therefore, from (9),

$$\sigma_{zj}^2 = (0.1143)^2 \times 144\left(\frac{144+12}{12}\right)S_j^2 + 1$$

From Appendix B and Table 12

$$W = \frac{12*(12-1)(0.653)}{(12-1)(9.99)} = \frac{86.196}{109.89} = 0.78$$

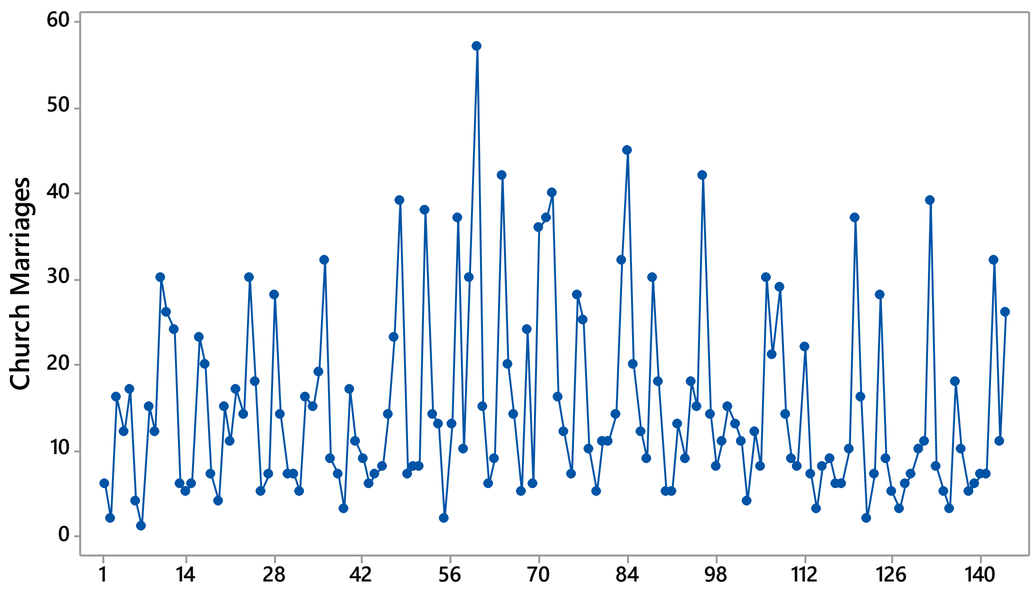

Fig. 1. Time plot of original series of church marriages

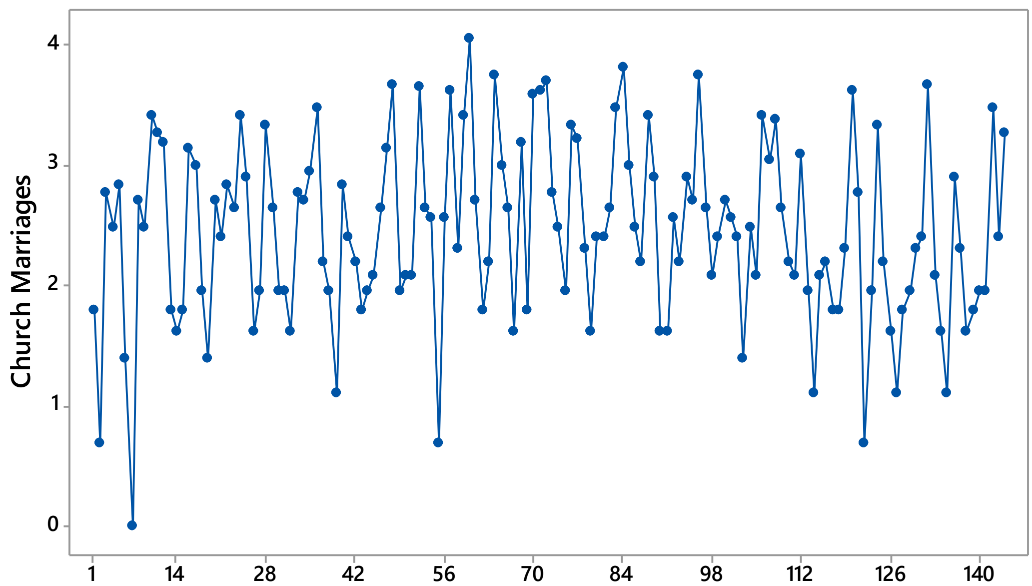

Fig. 2. Time plot of transformed series of church marriages

Table 9. Seasonal effects (S_j), estimate of the column variance ($\hat{\sigma}_j^2$) and calculated chi-square $\left(\chi_{cal}^2\right)$

j	1	2	3	4	5	6	7	8	9	10	11	12
S_j	1.65	0.76	0.76	2.16	1.16	0.53	0.31	0.66	0.62	0.97	0.97	1.62
$\hat{\sigma}_j^2$	24.6	10.7	11.8	82.2	28.4	13.2	4.3	27.6	73.1	101	81.6	84.6
χ_{cal}^2	0.06	0.08	0.09	0.09	0.08	0.19	0.17	0.18	0.55	0.33	0.34	0.16

Table 10. Transformed data of deviations of the observed values from averages

($Z_{ij} = \left| y_{ij} - \bar{y}_{,j} \right|$)

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total	$\bar{z}_{i.}$	$\sigma_{i.}$
2008	0.64	1.08	0.81	0.68	0.20	0.56	1.42	0.37	0.19	0.57	0.32	0.40	7.22	0.60	0.36
2009	0.64	0.16	0.17	0.03	0.36	0.00	0.03	0.37	0.11	0.00	0.30	0.17	2.35	0.20	0.19
2010	0.46	0.16	0.02	0.15	0.00	0.00	0.53	0.73	0.48	0.12	0.00	0.11	2.79	0.23	0.25
2011	0.24	0.17	0.86	0.31	0.24	0.25	0.38	0.40	0.21	0.19	0.19	0.09	3.55	0.30	0.20
2012	0.49	0.31	0.12	0.45	0.00	0.62	0.72	0.22	1.32	0.53	0.46	0.47	5.74	0.48	0.34
2013	0.27	0.02	0.23	0.55	0.36	0.70	0.19	0.84	0.50	0.75	0.67	0.12	5.23	0.44	0.27
2014	0.34	0.71	0.02	0.19	0.58	0.36	0.19	0.06	0.11	0.19	0.52	0.23	3.49	0.29	0.22
2015	0.56	0.71	0.23	0.22	0.26	0.33	0.19	0.22	0.09	0.06	0.23	0.16	3.31	0.28	0.19
2016	0.20	0.31	0.44	0.46	0.07	0.46	0.03	0.14	0.21	0.57	0.10	0.21	3.19	0.27	0.18
2017	0.20	0.42	0.12	0.05	0.69	0.84	0.66	0.14	0.50	1.04	0.64	0.04	5.37	0.45	0.33
2018	0.34	1.08	0.02	0.15	0.44	0.33	0.32	0.55	0.35	0.53	0.54	0.09	4.75	0.40	0.28
2019	0.36	0.16	0.86	0.26	0.33	0.33	0.38	0.40	0.35	0.64	0.54	0.32	4.93	0.41	0.19
Total	4.74	5.30	3.90	3.62	3.53	4.80	5.04	4.43	4.42	5.19	4.53	2.40	51.91		
$\bar{z}_{.j}$	0.40	0.44	0.32	0.30	0.29	0.40	0.42	0.37	0.37	0.43	0.38	0.20		0.36	
$\sigma_{.j}$	0.16	0.37	0.34	0.20	0.21	0.25	0.38	0.24	0.34	0.32	0.22	0.13			0.27

Table 11. Transformed series of square of deviations of the observed values from seasonal averages

$\left(z_{ij} - \bar{z}_{.j} \right)^2$

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	total
2008	0.06	0.41	0.24	0.14	0.01	0.02	0.99	0.00	0.03	0.02	0.00	0.04	1.96
2009	0.06	0.08	0.02	0.08	0.00	0.16	0.15	0.00	0.07	0.18	0.01	0.00	0.81
2010	0.00	0.08	0.09	0.02	0.08	0.16	0.01	0.13	0.01	0.10	0.14	0.01	0.84
2011	0.02	0.07	0.29	0.00	0.00	0.02	0.00	0.00	0.02	0.06	0.03	0.01	0.54
2012	0.01	0.02	0.04	0.03	0.08	0.05	0.09	0.02	0.91	0.01	0.01	0.07	1.34
2013	0.01	0.18	0.01	0.08	0.00	0.09	0.05	0.22	0.02	0.10	0.08	0.01	0.85
2014	0.00	0.07	0.09	0.02	0.08	0.00	0.05	0.10	0.07	0.06	0.02	0.00	0.57
2015	0.03	0.07	0.01	0.00	0.00	0.00	0.05	0.02	0.07	0.14	0.02	0.00	0.43
2016	0.04	0.02	0.01	0.02	0.05	0.00	0.15	0.05	0.02	0.02	0.08	0.00	0.47
2017	0.04	0.00	0.04	0.05	0.16	0.20	0.06	0.05	0.02	0.37	0.07	0.03	1.08
2018	0.00	0.41	0.09	0.02	0.02	0.00	0.01	0.03	0.00	0.01	0.03	0.01	0.64
2019	0.002	0.08	0.29	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.03	0.01	0.46
Total	0.28	1.48	1.24	0.46	0.50	0.71	1.63	0.63	1.24	1.10	0.52	0.19	9.99

Table 12. Calculation of $m\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$ m =13

$\bar{z}_{.j}$	$\bar{z}_{..}$	$\bar{z}_{.j}-\bar{z}_{..}$	$\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$	$12\times\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$
0.4	0.36	0.040	0.002	0.019
0.44	0.36	0.080	0.006	0.077
0.32	0.36	-0.040	0.002	0.019
0.3	0.36	-0.060	0.004	0.043
0.29	0.36	-0.070	0.005	0.059
0.4	0.36	0.040	0.002	0.019
0.42	0.36	0.060	0.004	0.043
0.37	0.36	0.010	0.000	0.001
0.37	0.36	0.010	0.000	0.001
0.43	0.36	0.070	0.005	0.059
0.38	0.36	0.020	0.000	0.005
0.2	0.36	-0.160	0.026	0.307
				0.653

4. SUMMARY, CONCLUSION AND RECOMMENDATION

This study has discussed Statistical properties of Buys-Ballot estimates for multiplicative model with their error terms. The method adopted in this study is Buys-Ballot procedure developed for choice of model and estimation trend parameters and seasonal indices among other uses based on row, column and overall averages and variances of the Buys- Ballot table. This study is limited to a series in when trend-cycle component is linear and admits multiplicative model. Properties of Buys-Ballot estimates discussed in details are used for (1) estimation of trend parameters (2) estimation of seasonal effect (3) choice of appropriate model for decomposition. The use of Buys-Ballot table for estimation and computation of trend parameters and seasonal effect was discussed in details. Successful transformation was applied for variance stability and normality of the time series data. The choice of model for decomposition is based on the seasonal variance of the Buys-Ballot table. The results indicate that (1) the column variance ($\hat{\sigma}_j^2$) of the Buys-Ballot depends on the seasonal indices (S_j^2) of the jth seasons for multiplicative model, (2) the suitable model that best describe the pattern in the transformed series is additive. This further confirms that the suitable model of the actual time series data is multiplicative. There is indication that choice of suitable model may be affected by violation of underlying assumptions, therefore, it is recommended that a study data should be evaluated for normality assumptions in the distribution, before applying test for choice of suitable time series model.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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APPENDIX

Appendix A. Original series on number of church marriages

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2013	16	34	21	12	23	24	14	27	13	11	39	27	21.75	80.93
2014	16	24	20	15	11	26	23	18	22	13	28	23	20.08	25.54
2015	8	18	11	14	26	14	24	18	21	19	25	10	17.33	36.24
2016	14	14	18	22	14	15	26	14	19	16	24	20	18.0	19.8
2017	17	21	20	23	16	11	27	22	14	12	5	21	17.42	37.72
2018	8	11	14	23	32	22	13	20	25	13	19	15	17.92	46.81
2019	5	21	9	10	18	15	9	20	18	10	15	16	13.83	25.95
2020	9	18	11	12	12	13	10	27	9	18	18	14	14.25	27.30
2021	6	10	10	11	18	12	11	10	11	8	11	10	10.67	7.88
2022	5	13	9	14	13	8	18	20	9	10	17	10	12.17	20.15
$\bar{X}_{.j}$	10.4	18.4	14.3	15.6	18.5	16.0	17.5	19.6	16.1	13.0	20.1	16.6	16.34	
$\sigma^2_{.j}$	23.38	51.38	24.46	26.04	50.78	35.56	47.39	27.16	32.22	13.11	90.1	35.6		41.37

Source: St Jude Parish, Amuzi Ahiara Mbaise (2013-2022)

Appendix B. Transformed series on number of church marriages

Year	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sept.	Oct.	Nov.	Dec.	$\bar{X}_{i.}$	$\sigma^2_{i.}$
2013	2.80	3.50	3.00	2.50	3.10	3.20	2.60	3.30	2.60	2.40	3.70	3.30	3.00	0.18
2014	2.80	3.20	2.30	2.70	2.60	3.30	3.10	2.90	3.10	2.60	3.30	3.10	2.90	0.11
2015	2.10	2.90	2.40	2.60	3.30	2.60	3.20	2.90	3.00	2.90	3.20	2.30	2.80	0.15
2016	2.60	2.60	2.90	3.20	2.60	2.70	3.30	2.60	2.90	2.80	3.20	2.30	2.80	0.08
2017	2.80	3.00	2.30	3.10	2.70	2.40	3.30	3.10	2.60	2.50	1.60	3.00	2.70	0.22
2018	2.10	2.40	2.60	3.10	3.50	3.10	2.60	2.30	3.20	2.60	2.90	2.70	2.80	0.17
2019	1.60	3.00	2.20	2.30	2.90	2.70	2.20	2.30	2.10	2.30	2.70	2.80	2.40	0.26
2020	2.20	2.90	2.40	2.50	2.50	2.60	2.30	3.30	2.20	2.90	2.90	2.60	2.60	0.21
2021	1.80	2.30	2.30	2.40	2.90	2.50	2.40	2.30	2.40	2.10	2.40	2.30	2.30	0.16
2022	1.60	2.60	2.20	2.60	2.60	2.10	2.90	2.30	2.20	2.30	2.80	2.30	2.40	0.22
$\bar{X}_{.j}$	2.20	2.90	2.50	2.70	2.90	2.70	2.80	2.70	2.60	2.50	2.90	2.70	2.70	
$\sigma^2_{.j}$	0.20	0.10	0.10	0.20	0.20	0.10	0.20	0.20	0.20	0.10	0.30	0.20		0.23

Source: St Jude Parish, Amuzi, Ahiara, Mbaise (2013-2022)

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