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Downlink MBER Transmit Beamforming Design Based on Uplink MBER Receive Beamforming for TDD-SDMA Induced MIMO Systems

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Abstract

The downlink minimum bit error rate (MBER) transmit beamforming is directly designed based on the uplink MBER receive beamforming solution for time division duplex (TDD) space-division multiple-access (SDMA) induced multiple-input multiple-output (MIMO) systems, where the base station (BS) is equipped with multiple antennas to support multiple single-antenna mobile terminals (MTs). It is shown that the dual relationship between multiuser detection and multiuser transmission can be extended to the rank-deficient system where the number of users supported is more than the number of transmit antennas available at the BS, if the MBER design is adopted. The proposed MBER transmit beamforming scheme is capable of achieving better performance over the standard minimum mean square error transmit beamforming solution with the support of low-complexity and high power-efficient MTs, particularly for rank-deficient TDD-SDMA MIMO systems. The robustness of the proposed MBER transmit beamforming design to the downlink and uplink noise or channel mismatch is investigated using simulation.

Keywords: Minimum Bit Error Rate, Time Division Duplex, Multiple-Input Multiple-Output, Transmit Beamforming, Receive Beamforming, Space-Division Multiple-Access

1. Introduction

Motivated by the demand for increasing throughput in wireless communication, antenna array assisted spatial processing techniques [1-7] have been developed in order to further improve the achievable spectral efficiency. In the uplink, the base station (BS) has the capacity to implement sophisticated receive (Rx) beamforming schemes to separate multiple user signals transmitted by mobile terminals (MTs). This provides a practical means of realising multiuser detection (MUD) for space-division multiple-access (SDMA) induced multiple-input multiple-output (MIMO) systems. Traditionally, adaptive Rx beamforming is based on the minimum mean square error (MMSE) design [2,5,6,8,], which requires that the number of users supported is no more than the number of receive antenna elements. If this condition is not met, the system becomes rank-deficient. Recently, adaptive minimum bit error rate (MBER) Rx beamforming design has been proposed [9-12], which outperforms the adaptive MMSE Rx beamforming, particularly in hostile rank-deficient systems.

In the downlink with non-cooperative MTs at the receive end, the mobile users are unable to perform sophisticated cooperative MUD. If the downlink's channel state information (CSI) is known at the BS, the BS can carry out transmit (Tx) preprocessing, leading to multiuser transmission (MUT). The assumption that the downlink channel impulse response (CIR) is known at the BS is valid for time division duplex (TDD) systems due to the channel reciprocity. However, for frequency division duplex systems, where the uplink and downlink channels are expected to be different, CIR feedback from the MT's receivers to the BS transmitter is necessary [13]. Many research efforts have been made to discover the equivalent relationship between the MUD and MUT [14-19]. Notably, Yang [20] has derived the exact equivalency between the MUD and MUT for TDD systems under the condition that the number of antennas at the BS is no less than the number of MTs supported[1].

[1]All the existing works, such as [14-20], consider the designs of MUD and/or MUT using second-order statistics based criteria, which implies that the MIMO system must have full rank.

According to the results of [20], the MUT can be obtained directly from MUD. Since the BS has to implement MUD, it can readily implement MUT based on its uplink MUD solution with no extra computational complexity cost. This is very attractive as this strategy enables the employment of low-complexity and high power-efficient MTs to achieve good downlink performance.

In general, the BS can design MUT when the downlink CSI is available. The Tx beamforming design based on the MMSE criterion is popular owing to its appealing simplicity [13,21]. Since the bit error rate (BER) is the ultimate system performance indicator, research interests in MBER based Tx beamforming techniques have intensified recently [22-25]. This MBER based MUT design invokes a constrained nonlinear optimisation [22-25], which is typically solved using the iterative gradient-based optimisation algorithm known as the sequential quadratic programming (SQP) [26]. However, the computational complexity of the SQP based MBER MUT solution can be excessive and may become impractical for high-rate systems [23]. This contribution adopts a very different approach to design the MBER Tx beamforming for TDD-SDMA induced MIMO systems, which does not suffer the above mentioned difficulty of high complexity. We prove that Yang's results [20] are more general and the exact equivalency between the MUD and MUT is not restricted only to second-order statistics based designs. Therefore, we can apply the results of [20] to implement the MBER Tx beamforming scheme directly based on the MBER Rx beamforming solution already available at the BS with no computational cost at all, even for rank-deficient systems where the number of antennas at the BS is less than the number of MTs supported. The robustness of the proposed scheme is also investigated when the downlink and uplink noise powers or channels mismatch.

2. Multiuser Beamforming System

The TDD-SDMA induced MIMO system considered is depicted in **Figure 1**, where the BS employs L antennas to support K single-antenna MTs. When the uplink is considered, the received signal vector $\mathbf{x}_U = [x_{U,1}\ x_{U,2} \cdots x_{U,L}]^T$ at the BS is given by

$$\mathbf{x}_U = \mathbf{H}\mathbf{s} + \mathrm{n}_U = \sum_{k=1}^{K} \mathbf{h}_k s_k + \mathrm{n}_U \qquad (1)$$

where the $L \times K$ channel matrix is given by $\mathbf{H} = [\mathbf{h}_1\ \mathbf{h}_2 \cdots \mathbf{h}_K]$, $\mathbf{h}_k = A_k \mathbf{g}_k$ with A_k and $\mathbf{g}_k$ denoting the channel coefficient and the steering vector for the kth user, respectively, $\mathbf{s} = [s_1\ s_2 \cdots s_K]^T$ contains the K data symbols transmitted by the K MTs to the BS, and n_U denotes the uplink channel

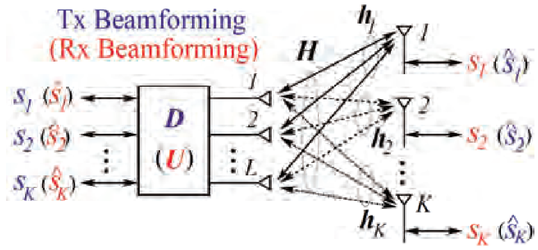

Figure 1. Schematic diagram of the TDD-SDMA induced MIMO system employing transmit and receive beamformings at the BS. The system employs L antennas at the BS to support K single-antenna MTs.

additive white Gaussian noise (AWGN) vector with $E[\mathrm{n}_U \mathrm{n}_U^H] = 2\sigma_U^2 \mathbf{I}_L$, and $\mathbf{I}_L$ represents the $L \times L$ identity matrix. Without the loss of generality, we assume the binary phase shift keying (BPSK) modulation. The result however can be extended to modulation schemes with multiple bits per symbol by adopting the minimum symbol-error-rate design [9]. The MUD at the BS consists of a bank of Rx beamformers

$$y_{U,k} = \mathrm{u}_k^H \mathbf{x}_U, 1 \le k \le K, \qquad (2)$$

where u_k is the Rx beamformer's weight vector for user k and H denotes the Hermitian operator. The decision variable vector $\mathbf{y}_U = [y_{U,1}\ y_{U,2} \cdots y_{U,K}]^T$ for the K transmitted symbols can be expressed as

$$\mathbf{y}_U = \mathbf{U}^H \mathbf{x}_U = \mathbf{U}^H \mathbf{H}\mathbf{s} + \mathbf{U}^H \mathrm{n}_U \qquad (3)$$

with the $L \times K$ Rx beamforming coefficient matrix expressed by

$$\mathbf{U} = [\mathrm{u}_1\ \mathrm{u}_2 \cdots \mathrm{u}_K]. \qquad (4)$$

The real part of $\mathbf{y}_U$ is a sufficient statistics for detecting $\mathbf{s}$.

The BS employs the Tx beamforming for the downlink transmission to the K MTs, with the $L \times K$ transmission preprocessing matrix

$$\mathbf{D} = [\mathrm{d}_1\ \mathrm{d}_2 \cdots \mathrm{d}_K], \qquad (5)$$

where $\mathbf{d}_k$ is the precoder's coefficient vector for preprocessing the symbol s_k to be transmitted to the k th MT. Note that we use the same notation $\mathbf{s}$ to represent the downlink symbol vector, without distinction from the uplink symbol vector for the purpose of notational simplification. Due to the reciprocity of the downlink and uplink channels, the received signal vector $\mathbf{y}_D = [y_{D,1}\ y_{D,2} \cdots y_{D,K}]^T$, received by the K MTs, is expressed as

$$\mathbf{y}_D = \mathbf{H}^T \mathbf{D}\mathbf{s} + \mathrm{n}_D, \qquad (6)$$

where n_D is the downlink AWGN vector with $E[\mathrm{n}_D \mathrm{n}_D^H] = 2\sigma_D^2 \mathbf{I}_K$ and ^{T} denotes the transpose operator. The real part of the decision variable $y_{D,k}$ is used by the k th MT for detecting the symbol s_k transmitted from the BS to the k th MT.

Under the condition $L \ge K$, there exists an exact equivalency between the Tx beamforming preprocessing matrix $\mathbf{D}$ and the Rx beamforming weight matrix $\mathbf{U}$ expressed by [20]

$$\mathbf{D} = \mathbf{U}^* \Lambda, \tag{7}$$

where * denotes the conjugate operator, $\Lambda = diag\{\lambda_1, \lambda_2, \cdots, \lambda_K\}$ is for achieving the transmit power constraint, and $\mathbf{U}$ or $\mathbf{D}$ is assumed to have been designed based on some second-order statistic criterion. The exact relationship (7) is valid for $\sigma_D^2 = \sigma_U^2$. A simple scheme to implement the transmit power constraint is to set $\lambda_k = 1/\|\mathbf{u}_k\|$ for $1 \le k \le K$ [20]. The relationship (7) is easy to understand. Under the condition of $L \ge K$, $\mathbf{U}^H\mathbf{H}$ of (3) and $\mathbf{H}^T\mathbf{D}$ of (6) have the same full rank and the same second-order statistic properties, given (7).

For rank-deficient systems where $L < K$, $\mathbf{U}^H\mathbf{H}$ and $\mathbf{H}^T\mathbf{D}$ no longer have the full rank. Indeed, both the MMSE Rx beamforming and MMSE Tx beamforming turn out to be deficient in this case, exhibiting a high BER floor. However, the MBER Rx beamforming scheme [10,12] has been shown to consistently outperform the MMSE design and is capable of operating in rank-deficient systems where the number of MTs is more than the number of receive antennas at the BS. We will show in the next section that the relationship (7) is not restricted only to second-order statistics based designs. Specifically, with the MBER MUD and MUT designs, (7) is valid and, moreover, the restriction $L \ge K$ is no longer required. This enables us to use (7) directly for designing the MBER Tx beamforming based on the available MBER Rx beamforming solution even for rank-deficient systems.

3. MBER Receive and Transmit Beamforming

The BER of detecting s_k using the uplink Rx beamforming with the weight vector $\mathbf{u}_k$ can be shown to be [10, 12]

$$P_{Rx,k}(\mathbf{u}_k) = \frac{1}{N_s}\sum_{q=1}^{N_s} Q\left(\frac{sgn(s_k^{(q)})\Re[\mathbf{u}_k^H \mathbf{H}\mathbf{s}^{(q)}]}{\|\mathbf{u}_k\|\sigma_U}\right), \tag{8}$$

where $Q(\bullet)$ is the usual Gaussian error function, $\Re[\bullet]$ denotes the real part, $N_s = 2^K$ is the number of all the equiprobable legitimate transmit symbol vectors, $\mathbf{s}^{(q)}$ for $1 \le q \le N_s$, and $s_k^{(q)}$ denotes the k th element of $\mathbf{s}^{(q)}$. The MBER solution for $\mathbf{u}_k$ is then defined as

$$\mathbf{u}_{\text{MBER},k} = \arg\min_{\mathbf{u}_k} P_{Rx,k}(\mathbf{u}_k). \tag{9}$$

The optimisation (9) can be solved using a gradient-based numerical optimisation algorithm [12]. Note that the BER is invariant to a positive scaling of $\mathbf{u}_k$, and one can always normalise the beamforming weight vector to a unit length, yielding $\|\mathbf{u}_k\| = 1$, which significantly reduces the computational complexity of optimisation. This is also useful for directly implementing the Tx beamforming design from the Rx beamforming design using the relationship (7), as the scaling matrix Λ can be chosen as the identity matrix in this case. Adaptive MBER Rx beamforming can be achieved using the stochastic gradient-based algorithm known as the least bit error rate [10, 12]. It is clear from (8) and (9) that the optimal MBER Rx beamforming solution to $\mathbf{u}_k$ is self-centred without regarding the effect on the other users. This type of optimisation is referred to as the egocentric-optimisation (E-optimisation) and the resul ting solution is known to be egocentric-optomum (E-opti -mum) [20]. The average BER over all the K users for the Rx beamforming with the beamforming weight matrix $\mathbf{U}$ is obviously given by

$$P_{Rx}(\mathbf{U}) = \frac{1}{K}\sum_{k=1}^{K} P_{Rx,k}(\mathbf{u}_k) = \frac{1}{KN_s}\sum_{k=1}^{K}\sum_{q=1}^{N_s} Q\left(\frac{sgn(s_k^{(q)})\Re[\mathbf{u}_k^H \mathbf{H}\mathbf{s}^{(q)}]}{\|\mathbf{u}_k\|\sigma_U}\right), \tag{10}$$

and the MBER Rx beamforming solution

$$\mathbf{U}_{\text{MBER}} = \arg\min_{\mathbf{U}} P_{Rx}(\mathbf{U}) \tag{11}$$

is simply given by $\mathbf{U}_{\text{MBER}} = [\mathbf{u}_{\text{MBER},1}\ \mathbf{u}_{\text{MBER},2} \cdots \mathbf{u}_{\text{MBER},K}]$. Since all the Rx beamforming vectors $\mathbf{u}_{\text{MBER},k}$ for $1 \le k \le K$ are optimum in some sense (E-optimum), the Rx beamforming matrix $\mathbf{U}_{\text{MBER}}$ is overall optimum (O-optimum) [20].

With the precoder coefficient matrix $\mathbf{D}$, the BER of detecting s_k by the k th MT is

$$P_{Tx,k}(\mathbf{D}) = \frac{1}{N_s}\sum_{q=1}^{N_s} Q\left(\frac{sgn(s_k^{(q)})\Re[\mathbf{h}_k^T \mathbf{D}\mathbf{s}^{(q)}]}{\sigma_D}\right) \tag{12}$$

for $1 \le k \le K$. The average BER over all the K users for the Tx beamforming with the precoder's weight matrix $\mathbf{D}$ is then given by

$$P_{Tx}(\mathbf{D}) = \frac{1}{K}\sum_{k=1}^{K} P_{Tx,k}(\mathbf{D}) = \frac{1}{KN_s}\sum_{k=1}^{K}\sum_{q=1}^{N_s} Q\left(\frac{sgn(s_k^{(q)})\Re[\mathbf{h}_k^T \mathbf{D}\mathbf{s}^{(q)}]}{\sigma_D}\right). \tag{13}$$

The MBER Tx beamforming solution for $\mathbf{D}$ can be obtained by minimising $P_{Tx}(\mathbf{D})$ subject to the given transmit power constraint

$$\mathbf{D}_{\text{MBER}} = \arg\min_{\mathbf{D}} P_{Tx}(\mathbf{D}) \quad \text{s.t. transmit power constraint is met} \tag{14}$$

This constrained nonlinear optimisation problem for example can be solved using the SQP algorithm [22, 23, 25], which is however computationally expensive. Unlike the E-optimum of the MBER Rx beamforming, the optimal MBER soultion to a precoder's column vector $\mathbf{d}_k$ is not self-centred, as it is clear from (12) to (14) that an optimal solution to $\mathbf{d}_k$ of user k not only maximises the k th user's performance but also pays attention on mitigating its effect on the other $K-1$ users. This type of optimisation is referred to as the altruistic-optimisation (A-optimisation) and the resulting solution is known to be altruistic-optimum (A-optimum) [20]. Denote $\mathbf{D}_{\text{MBER}} = [\mathbf{d}_{\text{MBER},1}\ \mathbf{d}_{\text{MBER},2} \cdots \mathbf{d}_{\text{MBER},k}]$. Since all the columns $\mathbf{d}_{\text{MBER},k}$ for $1 \le k \le K$ are optimum in some sense (A-optimum), the precoding matrix $\mathbf{D}_{\text{MBER}}$ is also O-optimum [20].

Instread of applying the computationally expensive SQP algorithm to find the MBER Tx beamforming solution, we can directly derive it as

$$\mathbf{D}_{\text{MBER}} = \mathbf{U}^*_{\text{MBER}}, \tag{15}$$

with no computational cost at all, provided that the relation (7) is not restricted to second-order statistics based designs and it is also valid for the MBER design. We now show that (7) is indeed much more general. We start by examining the probability density functions (PDFs) of $\Re[\mathbf{y}_D]$ and $\Re[\mathbf{y}_u]$, the real part of $\mathbf{y}_D$ in (6) and the real part of $\mathbf{y}_U$ in (3), respectively. Without loss of generality, we assume that $\mathbf{u}_k^H\mathbf{u}_k = 1$, $1 \le k \le K$, as the BER is invariant to the length of $\mathbf{u}_k$ [12]. Denote $\mathbf{H} = \mathbf{H}_R + j\mathbf{H}_I$, with the real part $\mathbf{H}_R = \Re[\mathbf{H}]$, the imaginary part $\mathbf{H}_I = \Im[\mathbf{H}]$ and $j^2 = -1$. Similarly, let $\mathbf{U} = \mathbf{U}_R + j\mathbf{U}_I$ and $\mathbf{n}_U = \mathbf{n}_{U_R} + j\mathbf{n}_{U_I}$. Then

$$\Re[\mathbf{y}_U] = (\mathbf{U}_R^T\mathbf{H}_R + \mathbf{U}_I^T\mathbf{H}_I)\mathbf{s} + (\mathbf{U}_R^T\mathbf{n}_{U_R} + \mathbf{U}_I^T\mathbf{n}_{U_I}). \tag{16}$$

The PDF of $\Re[\mathbf{y}_U]$, denoted as $p_U(w_1, w_2, \cdots, w_K)$, is obviously Gaussian with the mean

$$E[\Re[\mathbf{y}_U]] = (\mathbf{U}_R^T\mathbf{H}_R + \mathbf{U}_I^T\mathbf{H}_I)\mathbf{s} \tag{17}$$

and the covariance matrix

$$Cov[\Re[\mathbf{y}_U]] = \begin{bmatrix} \sigma_U^2 & \sigma_U^2\Re[\mathbf{u}_1^H\mathbf{u}_2] & \cdots & \sigma_U^2\Re[\mathbf{u}_1^H\mathbf{u}_K] \\ \sigma_U^2\Re[\mathbf{u}_2^H\mathbf{u}_1] & \sigma_U^2 & \ddots & \sigma_U^2\Re[\mathbf{u}_2^H\mathbf{u}_K] \\ \vdots & \ddots & \ddots & \cdots \\ \sigma_U^2\Re[\mathbf{u}_K^H\mathbf{u}_1] & \sigma_U^2\Re[\mathbf{u}_K^H\mathbf{u}_2] & \cdots & \sigma_U^2 \end{bmatrix}. \tag{18}$$

On the other hand, with the notation $\mathbf{n}_D = \mathbf{n}_{D_R} + j\mathbf{n}_{D_I}$ and given $\mathbf{D} = \mathbf{U}^*$,

$$\Re[\mathbf{y}_D] = (\mathbf{H}_R^T\mathbf{U}_R + \mathbf{H}_I^T\mathbf{U}_I)\mathbf{s} + \mathbf{n}_{D_R}. \tag{19}$$

The PDF of $\Re[\mathbf{y}_D]$, denoted as $p_D(w_1, w_2, \cdots, w_K)$, is Gaussian with the mean

$$E[\Re[\mathbf{y}_D]] = (\mathbf{H}_R^T\mathbf{U}_R + \mathbf{H}_I^T\mathbf{U}_I)\mathbf{s} \tag{20}$$

and the covariance matrix

$$Cov[\Re[\mathbf{y}_D]] = \sigma_D^2\mathbf{I}_K. \tag{21}$$

The BER (10) is determined by the K marginal PDFs of $\Re[\mathbf{y}_U]$, $p_{U,k}(w_k)$ for $1 \le k \le K$, which are Gaussian distributed and are specified by the mean vector (17) and the diagonal elements of the covariance matrix (18). The K marginal PDFs of $\Re[\mathbf{y}_D]$, $p_{D,k}(w_k)$ for $1 \le k \le K$, are also Gaussian distributed and are specified by the mean vector (20) and the diagonal elements of the covariance matrix (21). These two sets of the marginal PDFs are almost ``identical", given $\sigma_U^2 = \sigma_D^2$. The difference is that $p_{U,k}(w_k)$ only depends on the k th column vector of $\mathbf{U}$ while $p_{D,k}(w_k)$ depends on the entire matrix $\mathbf{U}^*$, leading to the BER expressions (10) and (13). We quote the following result from [20].

Proposition 1 *An E-optimum solution in a MUD is equivalent to an A-optimum solution in the corresponding MUT.*

Now let $\mathbf{U}_{\text{MBER}}$ be the MBER Rx Beamforming solution of (11). That is, the column vectors of $\mathbf{U}_{\text{MBER}}$ are E-optimum in the context of the MBER Rx Beamforming. Then $\mathbf{U}^*_{\text{MBER}}$ is the MBER Tx Beamforming solution of (14), and the column vectors of $\mathbf{U}^*_{\text{MBER}}$ are A-optimum in the context of the MBER Tx Beamforming. Note that we do not require $L \ge K$.

4. Simulation Study

Full-rank system. The BS employed a four-element linear antenna array with half-wavelength element spacing to support four single-antenna BPSK users. The angles of arrival (departure) for the four users were -2°, -16°, 15° and 30°, respectively, and the uplink channel coefficients for the four users were $A_k = 0.7071 + j0.7071$, $1 \le k \le 4$. The full uplink CSI was assumed to be known at the BS and was used by the BS to design the uplink Rx beamforming. **Figure 2** compares the average BER performance, defined in (10), of the uplink MMSE and MBER Rx beamforming schemes. The BS then directly implemented the down-

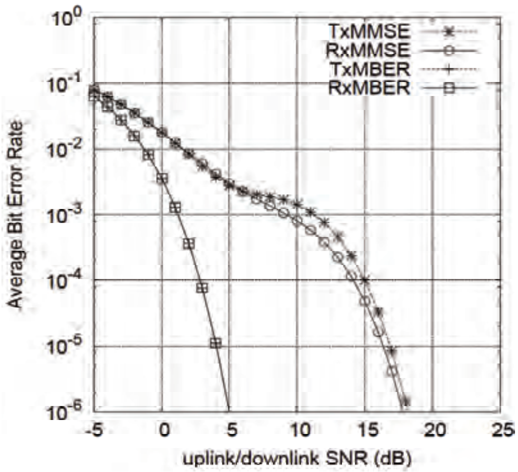

Figure 2. Average BER performance comparison of the uplink Rx and downlink Tx beamforming schemes with both the MMSE and MBER designs for the TDD-SDMA induced MIMO system which consists of a four-antenna BS to support four single-antenna BPSK MTs.

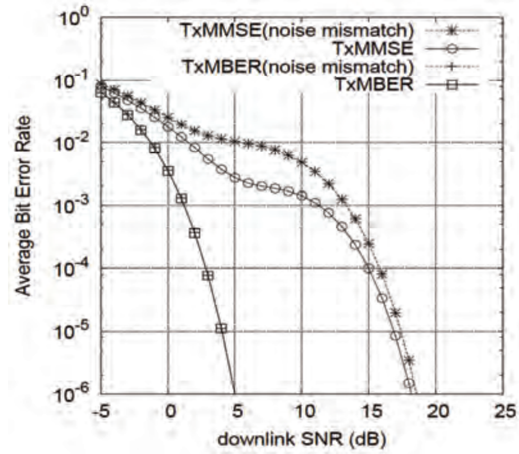

Figure 3. Average BER performance of the downlink Tx beamforming schemes with both the MMSE and MBER designs for the TDD-SDMA induced MIMO system which consists of a four-antenna BS to support four single-antenna BPSK MTs, when the downlink and uplink noise powers mismatch.

link Tx beamforming from the resulting uplink Rx beamforming according to (7). Assuming the exact reciprocity of the uplink and downlink channels as well as $\sigma_U^2 = \sigma_D^2$, the average BERs, as defined in (13), of both the MMSE and MBER Tx beamforming schemes so designed are also plotted in **Figure 2**, in comparison with the average BERs of the MBER and MMSE Rx beamforming designs. As expected, the performance of the proposed downlink Tx beamforming design agreed with that of the uplink Rx beamforming design.

The robustness of the proposed Tx beamforming design was next investigated when the downlink and uplink noise powers or channels were mismatched. In the case of noise mismatching, the downlink noise power was 3 dB more than the uplink noise power. The average BERs of the MBER and MMSE Tx beamforming designs under this uplink and downlink noise power mismatching are plotted in **Figure 3**, in comparison with the case of equal uplink and downlink noise powers. It can be seen that the 3 dB noise-power mismatching had little influence on the performance of the MBER Tx beamforming scheme but it had some influence on the performance of the MMSE Tx beamforming design. This was expected as the MMSE design is explicitly influenced by the noise power while the BER calculation is relatively insensitive to the noise variance estimate used. In the case of channel mismatching, the uplink channel coefficients were $A_k = 0.7071 + j0.7071$ for $1 \le k \le 4$, but the downlink channel coefficients were $A_k = 0.6 + j0.8$ for $1 \le k \le 4$. The average BER performance of the MBER and MMSE Tx beamforming designs under this uplink and downlink channel mismatching are plotted in **Figure 4**, in comparison with the case of the exact reciprocity of uplink and downlink channels. From **Figure 4**, it can be seen that this imperfect CSI had little influence on the MMSE Tx beamforming scheme. It is also seen that the MBER Tx beamforming design was not overly sensitive to the imperfect CSI.

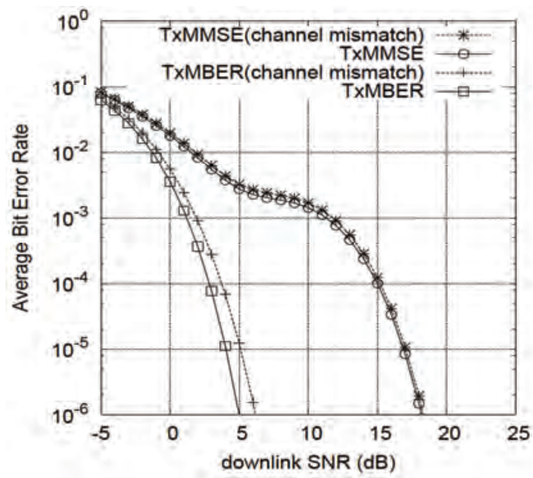

Figure 4. Average BER performance of the downlink Tx beamforming schemes with both the MMSE and MBER designs for the TDD-SDMA induced MIMO system which consists of a four-antenna BS to support four single-antenna BPSK MTs, when the downlink and uplink channels mismatch.

Rank-deficient system. The BS again employed a four-element linear antenna array with half-wavelength element spacing but the number of single-antenna BPSK users was increased to six. The angles of arrival (departure) for the six users were -2°, -15°, 10°, -30°, 25°

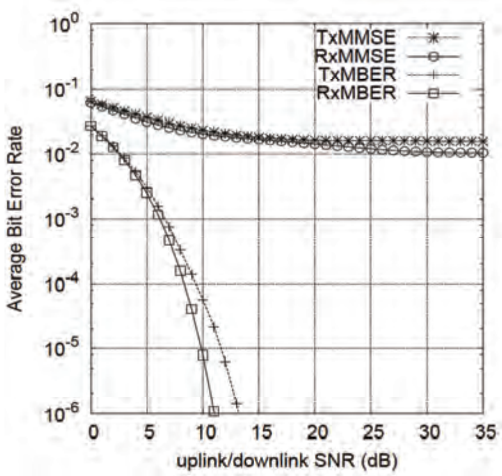

Figure 5. Average BER performance comparison of the uplink Rx and downlink Tx beamforming schemes with both the MMSE and MBER designs for the TDD-SDMA induced MIMO system which consists of a four-antenna BS to support six single-antenna BPSK MTs.

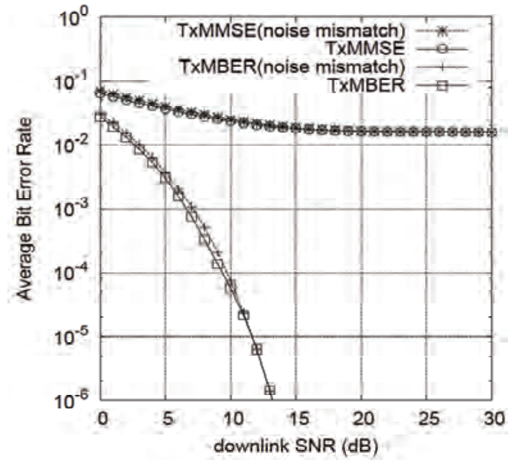

Figure 6. Average BER performance of the downlink Tx beamforming schemes with both the MMSE and MBER designs for the TDD-SDMA induced MIMO system which consists of a four-antenna BS to support six single-antenna BPSK MTs, when the downlink and uplink noise powers mismatch.

and 36°, respectively, and the uplink channel co-efficients for the six users were $A_k = 0.7071 + j0.7071$, $1 \le k \le 6$. Given the exact reciprocity of the uplink and downlink channels as well as $\sigma_U^2 = \sigma_D^2$, the average BERs of both the MBER and MMSE Tx beamforming designs, implemented directly from the corresponding Rx beamforming schemes according to (7), are plotted in **Figure 5**, in comparison with the average BERs of the MBER and MMSE Rx beamforming designs. For this rank-deficient system, both the MMSE Rx and Tx beamforming solutions exhibited similar high BER floors while both the MBER Rx and Tx beamforming solutions achieved similarly adequate performance.

The robustness of the proposed Tx beamforming design was then investigated again under the scenarios of mismatched downlink and uplink noise powers or channels. In the case of the downlink channel having 3 dB more noise power than the uplink channel, the average BERs of the MBER and MMSE Tx beamforming designs, implemented directly from the corresponding Rx beamforming solutions according to the relation (7), are plotted in **Figure 6**, in comparison with the case of $\sigma_U^2 = \sigma_D^2$. It can be seen that the 3 dB noise-power mismatching had little influence on performance. In the case of channel mismatching, the uplink channel coefficients were $A_k = 0.7071 + j0.7071$ for $1 \le k \le 6$, but the downlink channel coefficients were $A_k = 0.6 + j0.8$ for $1 \le k \le 6$. The average BER performance of the MBER and MMSE Tx beamforming designs under this uplink and downlink channel mismatching are plotted in **Figure 7**, in comparison with the case of an identical uplink and downlink channel.

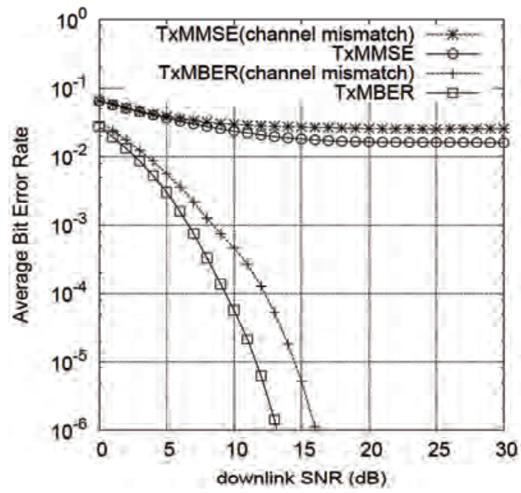

Figure 7. Average BER performance of the downlink Tx beamforming schemes with both the MMSE and MBER designs for the TDD-SDMA induced MIMO system which consists of a four-antenna BS to support six single-antenna BPSK MTs, when the downlink and uplink channels mismatch.

From **Figure 7**, it can be seen that the MBER Tx beamforming design was not overly sensitive to the imperfect CSI.

5. Conclusions

The downlink MBER transmit beamforming solution has been derived directly based on the uplink MBER receive beamforming design for TDD-SDMA induced MIMO systems. It has been shown that even for rank-deficient TDD systems, where the number of MTs supported is more than the number of transmit antennas available at the BS, the equivalent relationship between the MUD and the corresponding MUT is still valid, if the MBER design is adopted. The proposed MBER transmit beamforming design imposes no computational cost at all at the BS and is capable of achieving good downlink BER performance with the support of low-complexity and high power-efficient MTs. The robustness of this transmit beamforming scheme to the downlink and uplink noise or channel mismatching has been investigated by simulation.

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