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Proposed Nonparametric Tests for the Umbrella Alternative in a Mixed Design for Location and Scale

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

Aims: To develop and compare tests for a mixed design of a Randomized Complete Block Design (RCBD) with a Completely Randomized Design (CRD) with k populations in testing for the umbrella alternative with known peak, p, for both location and scale parameters. These tests were combinations of the Mack-Wolfe test, the Kim-Kim test, and both the Mack-Wolfe test, and the Kim-Kim test using the Moses technique.
Study Design: Monte Carlo Simulation Study.
Place and Duration of Study: North Dakota State University, Department of Statistics, August 2020-December 2021.
Methodology: A simulation study was conducted to see how well the proposed tests maintained their significant levels. Powers were also estimated for different ratios of sample size in the CRD to the number of blocks in the RCBD; we assumed equal variance ratios between the CRD and the RCBD. Different parameter changes were considered to see if they would impact which test statistics had greater power. In all cases, it was assumed three observations per treatment, per block in the RCBD portion.

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Conclusion: All proposed tests maintained their significance levels. An overall test is recommended if both location and scale parameters change.

Keywords: *Completely randomized design; randomized complete block design; known turning point; mixed design for location and scale*

1 Introduction

The nonparametric approach is sometimes preferred by researchers since it requires fewer assumptions than parametric tests for the results to be valid [1, pp.16-20]. There are times when researchers may want to test whether there is a difference in means or variances or both among several populations. In some of these cases, the researcher can assume that there is an umbrella effect among the means and variances if they are different. Namely, the means (or variances) may be nondecreasing up to a point, and then nonincreasing after that point, and where at least two are different [2]. A researcher may be able to assume this effect when increasing the dosages of a drug. The drug effect on the experimental unit may, at first, be increasing (or nondecreasing), but after a certain level, the drug effect on the experimental unit might start to decrease with increasing dosage of the drug. The test of hypotheses for the umbrella alternative testing for means and variances is given in equation (1):

$$H_0: \mu_1 = \cdots = \mu_k \text{ and } H_0: \sigma_1 = \cdots = \sigma_k$$
$$H_a: \mu_1 \leq \mu_2 \leq \cdots \leq \mu_p \geq \cdots \geq \mu_k \text{ and } H_a: \sigma_1 \leq \sigma_2 \leq \cdots \leq \sigma_p \geq \cdots \geq \sigma_k \tag{1}$$

with at least one strict inequality, where μ_i and σ_i represent the mean and variance (or scale parameter) of the ith population. The value, p, is called the peak of the umbrella. The means and variances are assumed to be nondecreasing up to the peak and nonincreasing on the other side of the peak. In this research, we developed tests for both the means and variances for the umbrella alternative in combination of an RCBD and a CRD mixed design. We defined the SB Ratio as the sample size in CRD to the number of blocks in the RCBD. SB ratios of 1/2, 1, and 2, and different underlying distributions were considered. We also estimated the powers of the tests that were developed when the means differed and the variances were equal, when the variances differed and the means were equal, and when both the means and variances differed.

In this section we present different proposed nonparametric test statistics related to a mixed design that can be used in testing for differences in means or variances.

1.1 Mack-Wolfe

The Mack-Wolfe test statistic [2] was designed for umbrella alternatives based on a CRD design. This test extends the Jonckheere and Terpstra test [3,4], which tests for the nondecreasing alternative, to the umbrella alternative hypothesis given in equation (1) for means. The test statistic, A_p for the case of known peak p, is the sum of Mann-Whitney [5] counts on either side of the peak and is given in (2).

$$A_p = \sum_{u=1}^{v-1} \sum_{v=2}^{p} U_{uv} + \sum_{u=\mathrm{p}}^{v-1} \sum_{v=p+1}^{k} U_{vu} \tag{2}$$

The null asymptotic distribution of the Mack-Wolfe (A_p) test statistic is normal. Under H_0, the mean and variance of A_p are given in equation (3):

$$\mathrm{E}_0(\mathrm{A_p}) = \frac{\mathrm{N_1^2+N_2^2-\sum_{i=1}^{k} n_i^2 - n_p^2}}{4} \tag{3}$$

$$var_0(A_p) = \frac{1}{72}\left\{2(N_1^3 + N_2^3) + 3(N_1^2 + N_2^2) - \sum_{i=1}^{k} n_i^2(2n_i + 3) - n_p^2(2n_p + 3) + 12n_pN_1N_2 - 12n_p^2N\right\}$$

where, $N_1 = \sum_{i=1}^{p} n_i$, $N_2 = \sum_{i=p}^{k} n_i$, and $N = N_1 + N_2 - n_p$, and n_p= the peak sample size.

Mack-Wolfe (1981) used the standardized test statistic A_p^* of the form

$$A_p^* = \frac{A_p - E(A_p)}{\sqrt{Var(A_p)}} \tag{4}$$

The null hypothesis is rejected for large values.

1.2 Kim-Kim

The Mack-Wolfe test was extended to a Randomized Complete Block Design (RCBD) by Kim-Kim [6]. The Kim-Kim statistic was formed by calculating the Mack-Wolfe test for each block and adding these up for all blocks. The Kim-Kim statistic is given in (5)

$$A = \sum_{j=1}^{b} A_{jp} \tag{5}$$

where, A_{jp} is the Mack-Wolfe test statistic for the j^{th} block, b is the number of blocks in the RCBD, k is the number of treatments. A_{ip} can be calculated using equation (2) for each block with i= 1, 2, ..., b.

The mean and variance of the Kim-Kim test statistic are given by

$$E_0(A) = \sum_{j=1}^{b} \left\{ \frac{\{N_{j1}^2 + N_{j2}^2 - \sum_{i=1}^{k} n_{ji}^2 - n_{jp}^2\}}{4} \right\}$$

$$var_0(A) = \frac{1}{72} \sum_{j=1}^{b} \left\{ 2\left(N_{j1}^3 + N_{j2}^3\right) + 3\left(N_{j1}^2 + N_{j2}^2\right) - \sum_{i=1}^{k} n_{ji}^2\left(2n_{ji} + 3\right) - n_{jp}^2\left(2n_{jp} + 3\right) + 12 n_{jp} N_{j1} N_{j2} - 12 n_{jp}^2 N_j \right\} \tag{6}$$

where, n_{ji} = sample size for i^{th} treatment in j^{th}block, n_{jp} = sample size for p^{th} treatment in j^{th}block, $N_{j1} = \sum_{i=1}^{p} n_{ji}$, $N_{j2} = \sum_{i=p}^{k} n_{ji}$, $N_j = N_{j1} + N_{j2} - n_{jp}$, In this research, we considered the case when n_{ji}= 1 (after applying the Moses technique when testing for variances). The mean and variance of A when n_{ji}= 1 (there is one observation per treatment for each block) to the form given below

$$E_0(A) = \frac{b\left(p^2 + (k-p+1)^2 - k - 1\right)}{4}$$

and

$$var_0(A) = \frac{b}{72}\begin{bmatrix} 2(p^3 + (k-p+1)^3) + 3(p^2 + (k-p+1)^2) - 5k \\ -5 + 12p(k-p+1) - 12k \end{bmatrix} \tag{7}$$

When H_0 is true, the asymptotic distribution of the standardized version is standard normal. The standardized version is given in (8).

$$A^* = \frac{A - E_0(A)}{\sqrt{var_0(A)}} \tag{8}$$

The null hypothesis is rejected for large values.

1.3 Alotaibi and Magel's Tests

Alotaibi and Magel [7] first extended the work of Magel, Terpstra, Canonizado & Park [8]. Magel et al. [8] proposed tests for the umbrella alternatives for means in an RCBD and a CRD mixed design. Alotaibi and Magel [7] considered different variance ratios between the error variance in the CRD to the error variance in the RCBD with the ratios being two, four, and nine. Powers were estimated for both tests when the sample size ratio in the CRD portion compared to the number of blocks in the RCBD portion, referred to as SB Ratio, was 1/8, 1/4, 1/3, 1/2, 1, 2, 3, 4, and 8. In all cases in the RCBD portion, it was assumed that there was one observation per treatment per block. They also assumed equal sample sizes for all treatments in the CRD portion.

Alotaibi and Magel [9] extended their work and developed three nonparametric tests for an umbrella alternative in a mixed design to test differences in means and variances. These tests were combinations of the Mack-Wolfe test for the means and a Moses Mack-Wolfe test for the variances. When calculating the Moses Mack-Wolfe test, the technique used in the Moses test [10] was applied before performing the Mack-Wolfe test. The Moses test was applied by randomly dividing up the original independent samples into m_i subsamples of equal size, q, $i = 1,2,..,k$ .For each of the m_i subsets, $i = 1,2,..,k$, the sample variance was calculated. The new observation set became the m_1 sample variances for the first sample treatment, the m_2 sample variances for the second sample treatment, and so on. The Mack-Wolfe test statistic was applied to this new set of observations.

The Moses Mack-Wolfe test statistic, MA_p, is the sum of Mann-Whitney counts on either side of the peak based on this new data set and is given in (9)

$$MA_p = \sum_{u=1}^{v-1}\sum_{v=2}^{p} U_{uv} + \sum_{u=\mathrm{p}}^{v-1}\sum_{v=p+1}^{k} U_{vu} \tag{9}$$

The mean, $E_0\big(MA_p\big)$, and variance, $var_0(MA_p)$, respectively, are given in (10).

$$E_0\big(MA_p\big) = \frac{M_1^2+M_2^2-\sum_{i=1}^{k} m_i^2-m_p^2}{4} \tag{10}$$

$$var_0(MA_p) = \frac{1}{72}\left\{2(M_1^3+M_2^3)+3(M_1^2+M_2^2) - \sum_{i=1}^{k} m_i^2(2m_i+3) - m_p^2\big(2m_p+3\big) + 12m_pM_1M_2 - 12m_p^2M\right\}$$

where $M_1 = \sum_{i=1}^{p} m_i$, $M_2 = \sum_{i=p}^{k} m_i$, and $M = \sum_{i=1}^{k} m_i = M_1 + M_2 - m_p$; m_i = the number of subsamples; and m_p= the number of subsamples in the peak.

Under the null hypothesis, the asymptotic distribution of the standardized version of the Moses Mack-Wolfe test statistic, MA_p^*, given in (11) is standard normal.

$$MA_p^* = \frac{MA_p - E(MA_p)}{\sqrt{Var(MA_p)}} \tag{11}$$

The null hypothesis is rejected for large values.

Their first proposed test, Z_1 is as shown in equation (12):

$$Z_1 = A_p^* + MA_p^* \tag{12}$$

where the Mack-Wolfe standardized version test of location for CRD is A_p^* as given in equation (4), and MA_p^* is the Moses Mack-Wolfe standardized version test of scale for CRD as given in equation (10). The null asymptotic distribution of Z_1 is normal with a mean of zero and a variance of 2. Their first proposed standardized version test, T_1, is given below in equation (13):

$$T_1 = \frac{Z_1 - 0}{\sqrt{2}} \tag{13}$$

T_1 has an asymptotic standard normal distribution when the null hypothesis is true. The null hypothesis is rejected for large values

Their second proposed test, Z_2 , for testing the hypotheses in equation (1) for a CRD is given in equation (14):

$$Z_2 = A_P + MA_P \tag{14}$$

where A_P is the Mack-Wolfe test for mean given in equation (2), and MA_P is the Moses Mack-Wolfe test for variance, as shown in equation (9). The mean and variance are given by:

$$E_0(Z_2) = E_0(A_P) + E_0(MA_P) \tag{15}$$

$$var_0(Z_2) = var_0(A_P) + var_0(MA_P) \tag{16}$$

The standardized version of their second proposed test (17) has an asymptotic normal distribution and the null hypothesis is rejected for large values.

$$T_2 = \frac{Z_2 - E_0(Z_2)}{\sqrt{var_0(Z_2)}} \tag{17}$$

They proposed a weighted standardized version of their second proposed test given in (18):

$$TW_2 = \frac{(A_P + 3 * MA_P) - E_0(A_P + 3 * MA_P)}{\sqrt{var_0(A_P + 9 * MA_P)}} \tag{18}$$

The null hypothesis is rejected for large values.

1.4 Lepage Test

Lepage's test [11] is a nonparametric test that tests for either differences in two variances or two means, or both. The test is a combination of the Mann-Whitney test for means [5] and the Ansari-Bradley test for variances [12]. The asymptotic null distribution of the Lepage test statistic is chi-square with two degrees of freedom.

1.5 Alsubie and Magel's Test

Alsubie, A., & Magel, R [13] proposed two nonparametric tests to test for differences in means and variances for the simple tree alternative in a CRD design. They conducted a simulation study that considered various conditions for three and four populations, and estimated powers as well as alpha values for the proposed tests. Situations were considered in which only the means were different, only the variances were different, and then when both means and variances were different among the populations.

Alsubie, A., & Magel, R. [14] proposed three additional nonparametric tests to test for the same case as above, but now using the Moses technique [10]. These test were compared with their previous tests on the basis of estimated powers [13].

2 Experimental Details / Methodology

In this research, we proposed three tests for testing the hypothesis given in (1) under a mixed design. The three tests are based on various combinations of the Mack-Wolfe [2]; Kim-Kim [6]; Moses Mack-Wolfe [9]; and Moses Kim-Kim using Moses's techniques (discussed in 2.1).

2.1 Moses Kim-Kim test

The Moses Kim-Kim test statistic will be for testing for the umbrella alternative for variances in an RCBD.

Within each block, observations are randomly divided into k samples of the same size, and each of these samples is randomly assigned a treatment within that block. In this research, the sample size for each treatment within a block was 3. The Moses technique is applied to this data. In our case, we had only one sample of size 3 associated with each treatment within a block. The sample variances of each of these samples of size 3 within a block were calculated. The new data set became the sample variances based on the subgroups with each treatment having only a sample of size 1 within a block. The Kim-Kim test statistic was calculated based on this transformed set data and referred to as the Moses Kim-Kim test statistic, MA. Therefore, the test statistic, MA, had the following form:

$$MA = \sum_{j=1}^{b} MA_{jp} \tag{19}$$

where b is the number of blocks in the RCBD where

MA_{jp} denotes the Moses Mack-Wolfe test statistic for the j^{th} block as follows:

$$MA_{jp} = \left\{ \sum_{u=1}^{v-1} \sum_{v=2}^{p} U_{juv} + \sum_{u=p}^{v-1} \sum_{v=p+1}^{k} U_{jvu} \right\}$$

When there is one observation per treatment per block, the expected value and the variance of the Moses-Kim-Kim test are in (20).

$$E_0(MA) = \frac{b\left(p^2+(k-p+1)^2-k-1\right)}{4} \tag{20}$$

and

$$var_0(MA) = \frac{b}{72}\left[2(p^3 + (k-p+1)^3) + 3(p^2 + (k-p+1)^2) - 5k - 5 + 12p(k-p+1) - 12k\right]$$

The asymptotic distribution of the standardized Moses Mack-Wolfe (21) test under the null distribution is the standard normal:

$$MA^* = \frac{MA - E_0(MA)}{\sqrt{var_0(MA)}} \tag{21}$$

The null hypothesis is rejected for large values.

We proposed three test statistics to test for the hypothesis in (1) for a mixed design (RCBD and CRD). The test statistics are combinations of the Mack-Wolfe test, the Moses Mack-Wolfe test, the Kim-Kim test, and the Moses Mack-Wolfe test.

2.2 Proposed test one

The first test statistic for the hypothesis in (1) is given in equation (22)

$$TK_1 = A_p^* + MA_p^* + A^* + MA^* \tag{22}$$

where A_p^* is the standardized Mack-Wolfe test based on the original data given in equation (4), and MA_p^* is the Moses Mack-Wolfe standardized version test of scale as given in equation (10). Also, $A^*$is the standardized Kim-Kim test based on the original data for RCBD given in equation (7), and MA^* is the Moses Kim-Kim standardized version test of scale for RCBD as given in equation (19). Therefore, the first proposed standardized version test, L_1, is given below in equation (23):

$$L_1 = \frac{TK_1 - 0}{\sqrt{4}} \tag{23}$$

Under H_0, L_1 has an asymptotic standard normal distribution. The null hypothesis is rejected if $L_1 \geq z_{\propto}$, where $z_{\propto}$ is the critical value for the upper-tail probability of the standard normal distribution. If the test is performed at a 5% level of significance, then $z_{\propto} = 1.645$.

2.3 Proposed test two

The second test statistic, TK_2, for testing the hypotheses in equation (1) is given in equation (24)

$$TK_2 = A_p + MA_p + A + MA \tag{24}$$

where, A_P is the Mack-Wolfe test (CRD) for location parameters, as shown in equation (2), and MA_P is the Moses Mack-Wolfe test (CRD) for scale as given in equation (9). Also, A is the Kim-Kim test statistic (RCBD) for location parameters, as shown in equation (5), and MA is the Moses Kim-Kim test statistic (RCBD) for scale as given in equation (19). The expected value and variance of TK_2 are the sum of the means and variances for the Mack-Wolfe tests for location, the Moses Mack-Wolfe test for scale, and the Kim-Kim tests for location, and the Moses Kim-Kim test for scale. The mean and variance are given in equations (25) and (26)

$$E_0(TK_2) = E_0(A_P) + E_0(MA_P) + E_0(A) + E_0(MA) \tag{25}$$

and

$$var_0(TK_2) = var_0(A_P) + var_0(MA_P) + var_0(A) + var_0(MA) \tag{26}$$

The standardized version of the second proposed test is given in equation (27)

$$L_2 = \frac{TK_2 - E_0(TK_2)}{\sqrt{var_0(TK_3)}} \tag{27}$$

Under H_0, L_2 has an asymptotic standard normal distribution. The null hypothesis is rejected for large values.

2.4 Proposed test three

The weighted standardized version of the second proposed test is the third proposed test and is given in equation (28):

$$LW_2 = \frac{((A_p + 3*MA_p) + (A + 3*MA)) - E_0((A_p + 3*MA_p) + (A + 3*MA))}{\sqrt{var_0(A_p + 9*MA_p) + var_0(A + 9*MA)}} \tag{28}$$

Under H_0, LW_2 has an asymptotic standard normal distribution. The null hypothesis is rejected for large values.

The idea behind proposing this test is that the sample size of the Moses Kim-Kim test is smaller than the sample size of the Kim-Kim test and therefore, more weight is applied to the Moses Kim-Kim test. In order to find Moses Kim-Kim test, the original sample must be divided into subsamples and the sum of the squared deviations found within each subsample. Since subsamples of size 3 were used in this study, the sample size used for the Moses Kim-Kim test was only 1/3 the sample size used for the Kim-Kim test. Hence, a weight of 3 was applied to the Moses Kim-Kim test.

2.5 Simulation study

A simulation study was conducted to compare the three proposed tests using SAS version 0.4 [15]/ Alpha values were first estimated for all of the tests to ensure that they were close to the stated alpha value used of 0.05. Powers were next estimated for each of the tests based on a variety of conditions. The design used was a mixed design of a CRD and RCBD. The underlying distributions used were normal, exponential, and t-distribution with three degrees of freedom. Three, four, and five populations were considered. For three populations, the peak was assumed to be 2. For four populations, the peaks considered were the second and third populations. In the case of five populations, the peaks considered were at the second, third, and fourth populations. Equal samples of 12 were taken from each of the k populations (n_1=n_2=…=n_k =n=12). Four subsets of 3 observations each were randomly formed from the 12 observations from each population, the sample variance of each of the subsets was calculated, and the Mack-Wolfe test and the Kim-Kim test were then calculated on these sample variances as well as on the original data.

The function RAND was used in SAS to generate samples from the distributions previously mentioned. The system clock was used as the seed.

The call function for the normal distribution is

F=RAND ('Normal', μ,σ)
*X=F *b + a*

The function (F) generated a random number from a normal distribution with the mean (μ) and the standard deviation (σ), respectively, and a and b were the change in location parameters and the change in scale parameters. The mean and standard deviation were 0 and 1, respectively. The values of a and b were initially set to 0 and 1, respectively.

The call function for the exponential distribution is

F=RAND ('Exponential', μ)
X=F +a

The function (F) generated a random number from an exponential distribution. The value a was used to adjust the location parameter appropriately. Initially, the value of a was set to 0. The location and scale parameters changed with different values of μ.

The call function for the t-distribution is

F=RAND ('T', 3*)*
*X=F * b + a*

This function generated a random number from a T-distribution with 3 degrees of freedom. Initially, the values of a and b were set to 0 and 1, respectively.

For all simulations, replications of 5,000 samples were used. The three proposed tests were compared in two parts. The first part of the simulation estimated the alpha values of the proposed test statistics. The stated alpha values for the proposed test statistics were all 0.05. The alpha values were estimated by counting the number of times the null hypothesis was rejected and then dividing that number by 5,000. The second part of the simulation study compared powers of the test statistics under various conditions. Powers were estimated by counting the number of times the proposed tests were rejected and then dividing that number by 5,000 for the various conditions.

2.6 Power calculations

In the cases of three, four, and five populations with the peak p assumed to be known, the means and variances that were considered were the following ($\mu_1, \mu_2, .., \mu_k$), (σ_1, σ_2, …, σ_k).

2.6.1 Three populations with peak at 2, and peak at 3

Powers were first estimated when all populations had the same variances and only the means changed. In this case, configurations of means were considered when one mean was the same as the peak mean, when there was equal spacing between the means directly on either side of the peak, and when there was unequal spacing between the means.

Similar powers were estimated when the means were the same, but the variances changed. Namely, powers were estimated when one other variance was equal to the peak variance, when there was equal spacing between the variances on the either side of the peak, and when there was unequal spacing among the variances on either side of the peak.

The powers were estimated when the means and variances differed among the populations

2.6.2 Four populations with peak at 2 and peak at 3

The powers were first estimated when the means could be different and the variances were all the same. In this case, configurations of the means were considered in which one mean was the same as the peak mean, there was equal spacing between means directly on either side of the peak, and there was unequal spacing between means directly on either side of the peak.

Similar situations were considered for the configuration of variances when the means were the same. Lastly, powers were estimated when the means and variances were different among the populations.

2.6.3 Five populations with peak at 2,and peak at 3 and peak at 4

Powers were first estimated when all populations had the same variances and only the means changed. In this case, configurations of means were considered when all means were distinct, when one mean was the same as the peak mean, when there was equal spacing between the means directly on either side of the peak, and when there was unequal spacing between the means.

Similar powers were estimated when the means were the same, but the variances changed. Namely, powers were estimated when one other variance was equal to the peak variance, when there was equal spacing between the variances on the either side of the peak, and when there was unequal spacing among the variances on either side of the peak. Lastly, powers were estimated when both the means and variances differed among the populations.

3 Results and Discussion

Selected results are given in Tables 1-7. The tables give results using various SB ratios and various underlying distributions. The SB Ratio was that the number of blocks in the RCBD portion divided by the sample size in the CRD portion. SB ratios were ½, 1, and 2 in this study Three situations of power estimates were considered in this study based on the parameter changes. The first situation considered was when at least some of the means were different, and the variances were equal. The second situation considered was when the means were equal, and at least some of the variances were different. The third situation considered was when at least some of the means and some of the variances were both different. Tables 1-3 show the estimated powers for 3 different types of populations when the locations (means) were different, and the scales (variances) were equal. Tables 4-6 show estimated powers when the locations (means) were the same and the scales (variances) were different. Tables 7-9 show estimated powers when both the locations (means) and the scales (variances) were different.

Table 1. Percentage of Rejection for k=3 Populations p=2; Normal Distributions with different means and equal variances when number of blocks half the sample size under mixed design. (n=12, Blk=6)

μ_1	${\sigma_1}^2$	μ_2	${\sigma_2}^2$	μ_3	${\sigma_3}^2$	L_1	L_2	LW_2
0	1	0	1	0	1	0.0518	0.0474	0.0534
0	1	1.5	1	0	1	0.9096	0.9922	0.9656
0	1	1.5	1	1.5	1	0.3978	0.5596	0.4600
1.5	1	1.5	1	0	1	0.3956	0.5672	0.4528
1.5	1	2	1	1.8	1	0.1816	0.2382	0.2048

Table 2. Percentage of Rejection for k=3 Populations p=2; T (3)-Distributions with different means and equal variances when number of blocks half the sample size under mixed design. (n= 12, Blk=6)

μ_1	${\sigma_1}^2$	μ_2	${\sigma_2}^2$	μ_3	${\sigma_3}^2$	L_1	L_2	LW_2
0	1 σ^2	0	1 σ^2	0	1 σ^2	0.0534	0.0496	0.0506
0	1 σ^2	1.5	1 σ^2	0	1 σ^2	0.7686	0.9256	0.8414
0	1 σ^2	1.5	1 σ^2	1.5	1 σ^2	0.3066	0.4268	0.3518
1.5	1 σ^2	1.5	1 σ^2	0	1 σ^2	0.3056	0.4238	0.3386
1.5	1 σ^2	2	1 σ^2	1.8	1 σ^2	0.1534	0.1948	0.1648

Table 3. Percentage of Rejection for k=3 Populations p=2; Exponential (1)-Distributions with different means and equal variance when number of blocks half the sample size under mixed design. (n= 12, Blk=6)

μ_1	${\sigma_1}^2$	μ_2	${\sigma_2}^2$	μ_3	${\sigma_3}^2$	$L1$	L_2	LW_2
1	1	1	1	1	1	0.0526	0.0472	0.0450
1	1	1.5	1	1	1	0.8682	0.9758	0.9254
1	1	1.5	1	1.5	1	0.3824	0.5092	0.4304
1.5	1	1.5	1	1	1	0.3854	0.5246	0.4400
1.5	1	2	1	1.8	1	0.6412	0.8152	0.7090

Table 4. Percentage of Rejection for k=3 Populations p=2; Normal Distribution with equal means and different variance when number of blocks equal the sample size under mixed design. (n=12, Blk=12)

μ_1	${\sigma_1}^2$	μ_2	${\sigma_2}^2$	μ_3	${\sigma_3}^2$	L_1	L_2	LW_2
0	1	0	1	0	1	0.0486	0.0546	0.0552
0	1	0	5	0	1	0.7914	0.2152	0.4980
0	1	0	5	0	5	0.3058	0.1056	0.1948
0	5	0	5	0	1	0.3140	0.1116	0.2062
0	5	0	9	0	8	0.2140	0.0868	0.1474

Table 5. Percentage of Rejection for k=3 Populations p=2; T (3)-Distribution with equal means and different variance when number of blocks equal the sample size under mixed design. (n= 12, Blk=12)

μ_1	${\sigma_1}^2$	μ_2	${\sigma_2}^2$	μ_3	${\sigma_3}^2$	L_1	L_2	LW_2
0	1 σ^2	0	1 σ^2	0	1 σ^2	0.0476	0.0476	0.0480
0	1 σ^2	0	5 σ^2	0	1 σ^2	0.7852	0.1414	0.1904
0	1 σ^2	0	5 σ^2	0	5 σ^2	0.2792	0.0790	0.0952
0	5 σ^2	0	5 σ^2	0	1 σ^2	0.2852	0.0834	0.1050
0	5 σ^2	0	9 σ^2	0	5 σ^2	0.3066	0.0924	0.1110

Table 6. Percentage of Rejection for k=3 Populations p=2; Exponential (1)-Distribution with equal means and different variance when number of blocks equal the sample size under mixed design. (n= 12, Blk=12)

μ_1	${\sigma_1}^2$	μ_2	${\sigma_2}^2$	μ_3	${\sigma_3}^2$	L_1	L_2	LW_2
1	1^2	1	1^2	1	1^2	0.0486	0.0432	0.0444
1	1^2	1	5^2	1	1^2	0.9966	0.2052	0.7248
1	1^2	1	5^2	1	5^2	0.6078	0.1054	0.2788
1	5^2	1	5^2	1	1^2	0.6178	0.1098	0.2856
1	5^2	1	9^2	1	8^2	0.2820	0.0718	0.1444

Table 7. Percentage of Rejection for k=3 Populations p=2; Normal Distribution with different means and different variance when number of blocks twice the sample size under mixed design. (n=12, Blk=24)

μ_1	${\sigma_1}^2$	μ_2	${\sigma_2}^2$	μ_3	${\sigma_3}^2$	L_1	L_2	LW_2
0	1	0	1	0	1	0.0486	0.0478	0.0496
0	1	1.5	5	0	1	0.9960	0.6264	0.8836
0	1	1.5	5	1.5	5	0.6844	0.2804	0.4458
1.5	5	1.5	5	0	1	0.6910	0.2824	0.4436
1.5	5	2	9	1.8	8	0.3124	0.1222	0.2038

Table 8. Percentage of Rejection for k=3 Populations p=2; T (3)-Distribution with different means and different variance when number of blocks twice the sample size under mixed design. (n= 12, Blk=24)

μ_1	${\sigma_1}^2$	μ_2	${\sigma_2}^2$	μ_3	${\sigma_3}^2$	L_1	L_2	LW_2
0	1 σ^2	0	1 σ^2	0	1 σ^2	0.0550	0.0536	0.0584
0	1 σ^2	1.5	5 σ^2	0	1 σ^2	1.0000	0.9962	0.9998
0	1 σ^2	1.5	5 σ^2	1.5	5 σ^2	0.9468	0.6494	0.8234
1.5	5 σ^2	1.5	5 σ^2	0	1 σ^2	0.9492	0.6496	0.8170
1.5	5 σ^2	2	9 σ^2	1.5	8 σ^2	0.5254	0.2884	0.3918

Table 9. Percentage of Rejection for k=3 Populations p=2; Exponential (1)-Distribution with different means and different variance when number of blocks twice the sample size under mixed design. (n= 12, Blk=24)

μ_1	${\sigma_1}^2$	μ_2	${\sigma_2}^2$	μ_3	${\sigma_3}^2$	L_1	L_2	LW_2
1	1^2	1	1^2	1	1^2	0.0452	0.0478	0.0484
1	1^2	2	2^2	1	1^2	1.0000	0.9576	0.9868
1	1^2	2	2^2	2	2^2	0.8870	0.5314	0.6186
2	2^2	2	2^2	1	1^2	0.8858	0.5362	0.6342
2	2^2	4	4^2	3	3^2	0.9926	0.7914	0.8814

4 Conclusion

All of the tests maintained their significance levels of 0.05. The results as to which test statistic had greater powers were the same for all the SB ratios considered and all distributions considered depending upon which set of parameters changed. For the three distributions considered with n= 12 and four subsets of 3 observations in each, number of populations either 3, 4, or 5, and various peaks, the L_2 test had the largest powers if only the means change. The L_1 test had the higher powers if only the variances change. The L_1 test had the highest powers if the means and variances change for some of the distributions. Overall, when researchers want to test for differences in both means and variances, L_1 test is recommended.

Competing Interests

Authors have declared that no competing interests exist.

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