Some Aspects of Blockchain-Enabled Radio Access Networks (B-RAN) Modeling: Review and Theoretical Study

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Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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Abstract

The impressive growth of the demand for wireless services for 5G and beyond, particularly encouraged by the Internet of Things, Internet of Everything, etc., raised up the importance of the problems related to the drastic increase (among others) of spectrum efficiency of the systems. In this regard two opportunistic trends were recently discussed by practical professionals and academicians: non-orthogonal spectrum sharing (NOMA transmission) for user equipment (UE) at physical layer level and decentralized Blockchain-enabled Radio Access (B-RAN) paradigm at network layer label. Both of them promise a significant growth on the transmission rate and the spectrum efficiency with low latency, etc. not only for 5G networks, but further, towards 6G networks and so on.

Moreover, it is a fact that the number of published works for B-RAN (some of them considered hereafter) is very impressive, however, the attention to its modeling aspects is rather limited.

The presented material is dedicated to present an approach for the practical small-scale modeling for the case of high-dimensional B-RAN based on continuous-time Markov processes with discrete set of states ($N \gg 1$) and self-similar processes for its traffic modeling.

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1 Introduction

The last decades clearly illustrate the fact that for the future wireless communications one of the most important subjects (certainly among others, which will drive considerable attention) is spectrum efficiency related issues. Recent trends to solve these problems can be stated as follows:

- Non-orthogonal signal transmission (Non-Orthogonal Multiple Access, NOMA) in general for user equipment (UE) at the physical layer of the system and
- Blockchain-enabled radio access by assembling the UE at the network layer.

It seems incorrect to contradict those trends, moreover it sounds more reasonable to consider them as "parts of a whole", i.e. "address" the problem to increase spectrum efficiency from different "directions", including also massive MIMO, etc.

Note, that NOMA and massive MIMO aspects will not be considered in the following as there is plenty of references for such topics (see, for example, [1] and references therein).

The number of references related to B-RAN is enormous (hundreds and maybe thousands), but for the following material only the topics of B-RAN modeling for which the number of publications is significantly limited were selected precisely. A brief overview of those selected topics together with the reasons (motivations) behind the ongoing material will be presented in the following.

A recent review of the blockchain enabled radio access network B-RAN can be found at [2] (see also references therein), where the authors announced their intention to create a "practical" B-RAN model, already published at [3]. The model presented there (called as a practical one and backed up with experimental results) was, to the best of the author's knowledge, really the first known attempt for a theoretic view for a B-RAN modeling approach. This material will be analyzed and commented in section II, and for future classification this kind of models will be denoted as B-RAN Markov models.

Let us present briefly, how the blockchain paradigm is applied at B-RAN. There are four steps for an admitting the request (valid request) to be included at the blockchain (see also [3]):

- Waiting to be included into a block;
- Waiting for confirmations;
- Waiting for service and finally;
- In service;

One can easily see that if at the third stage several requests (for the same block) arrive simultaneously, then some previous events can influence the current stage of the queue and so the process obviously is not a Markov one. If it is possible to neglect this scenario, for example, when the request flow is not so intense, then the Markov character of the queuing process can be successfully applied, as it was assumed at [3]. It is worth stressing here that the queuing model for B-RAN created in this way, might be "conditionally" addressed as a "service" model for B-RAN.

Here one "aspect" of the Markov models naturally raises up: being that such kind of queue modeling is well established, is it possible to apply it to B-RAN?

Another modeling "aspect" has to be dedicated to the traffic characterization in a mobile network with B-RAN. Sure, those models (traffic and service) are significantly "interconnected", but our attempt to analyze those mutual features has failed so far and it can only be achieved under the simplest assumptions, as it was done at [3-5 etc.] Though, it was found reasonable to analyze them separately and section III presents the traffic characterization using self-similar traffic models.

It is worth mentioning, that the problem of the adequate traffic modeling of mobile networks is not new as well! The related works with the most consistent results can be found in the papers [2, 3, 6-13] and the book [14]. For the traffic theory of the networks (see, for example [15,16]) all those works use not only Markov modeling but apply as well the theory of self-similar processes for interpretation of the traffic [17] in wireless communication

networks of 5G and beyond. In order to simplify the notations, in the following such models will be denoted as self-similar ones.

As a final comment related to modeling in this introduction let us be emphatic in saying that for practical modeling it is not only enough to formulate and proof the adequate theoretical model, but it is also of paramount importance to propose a working strategy that allows to synthesize the model based on the available stochastic data (a priori information for the modeling). All the motivations presented above will be addressed and detailed in the ongoing material.

The author is mindful, that some of the ongoing statements are not rigorously proofed and are illustrated only in a "physical" (quantitative) way but hopes that even in this form they can motivate the interested reader.

The paper is organized as follows. Section II is totally dedicated to Markov modeling for B-RAN. Basics for the description of traffic in terms of α-stable processes and a stochastic description of self-similar modeling for B-RAN traffic are presented in section III. Conclusions are presented in section IV. All necessary theoretical results applied in this paper are presented in the appendixes A1-A4.

2 Markov Modeling for B-RAN

As it was pointed out above, (and to the best of the author's knowledge) the first attempt to create Markov models for B-RAN was presented in [3] based on the following assumptions.

Assume i_n as a number of pending requests with "*n*" confirmations, but note, that each pending request is confirmed only after reception of "*N*" confirmations in order to be mined into queue, but not served yet. Its number is obviously equal to $\sum_{n=N}^{\infty} i_n$.

Though, the B-RAN is fully described by the state $X_j(i_0, i_1, ..., i_{N-1}, \sum_{n=N}^{\infty} i_n)$ which belongs to (*N*+1) dimensional state space S_+^{N+1}. Then simplifying dimensionality to (*N*=1) and approximating the request arrival process, the block generation process and the service time process (all of them), as Poisson processes with the corresponding intensities $\lambda^a, \lambda^b, \lambda^c$ and following the well-known approach from [15], it was possible to obtain finally the equations (7)-(9) and the state transition plot (Fig. 4) presented in [3].

Though, the practical model from [3] is completed as a time-homogenous (with discrete states), time-continuous Markov process and its application, verification and so on (with the assumption $\lambda^b > \lambda^a$) is thoroughly presented in [3].

Taking the latter inequality condition into account, an interesting question rises up naturally: for the conditions $\lambda^b > \lambda^a$ (considered as "practical" in [3]) what is the difference between the model presented in [3] and the theoretical well known process denoted as "birth-and-death" [15]. Simple comparison between them (particularly for condition $\lambda^c, \lambda^b > \lambda^a$) shows that they are almost the same and it follows directly from Fig. 4 in [3].

This statement is rather impressive, but not surprising, and allows making the following conclusion: for negligible values of the intensity of arrival request access comparing with λ^b and λ^c, (which are obviously at least sufficient conditions for the Markov modeling of B-RAN), the adequate model of the queue might be the birth-and-death process (see [15], etc.)

In the following this statement is assumed, but it has to be mentioned once more, that it follows directly as a result of the initial consideration of the Markov assumption for $X(i_0, i_1, ..., i_{N-1}, \sum_{n=N}^{\infty} i_n)$, made in [3] together with its further simplification (*N*=1).

At the same time one has to notice, that for security reasons in B-RAN the value "*N*" has to be much more than 1. Though, such kind of practical modeling as that presented in [3] (as well as "birth-and-death" model) can be applied only as a first approximation for the concrete assumptions, mentioned above. It is worth mentioning that at [3], some attempts to extend the application of formulas for the case *N*=1 for the general case of *N*>1 are presented, particularly for evaluating the average latency in B-RAN, etc.

Here it is time to stress, that even in the Markov framework for the process $X(i_0, i_1, ..., i_{N-1}, \sum_{n=N}^{\infty} i_n)$ constructive results can only be obtained for the one-confirmation assumption.

But the statement in [3] is true, that the goal in the following is to investigate the general *N*-confirmation problem, which is rather complex to achieve analytically due to the high dimensionality of the discrete space and the corresponding matrixes in the forward Kolmogorov-Chapman equations [15].

The latter comment absolutely does not mean, that the results obtained from *N*=1 models are not meaningful (see the material below) for providing practical modeling for B-RAN.

It seems reasonable to try to study *N*-dimensional models in a different way, for example, assuming the previously considered *N*-dimensional time-homogeneous models with discrete state space as prelimited case models for the continuous *N*-dimensional Markov processes which are sometimes more adequate to study the *N*-dimensional scenarios (when *N* is much more than 1).This assumption is typical in Markov theory: start with the discrete case as prelimited and follow with the continuous case as an asymptotic one (*N* much more,than1).

In this regard it is practical to make a "shift" on the B-RAN modeling from Markov models with continuous time and a *N* number of discrete states to the *N*-dimensional Markov models with continuous time and "continuous" states.

Hereafter, the *N*-dimensional diffusive Markov models (analytically described by stochastic differential equations, SDE of diffusive-isotropic type) were selected for analysis (see appendix A2).
What are the arguments for it?

1. The *N*-dimensional vector Markov process *x*(*t*) as a model for B-RAN can highly likely be characterized by its power spectrum for each component as "filling" the whole frequency axis and not as concentrated at fixed "regions" of the frequency axis.

Though, its covariation function in stable-state conditions is a monotonic function of "τ", i.e.

$$B_x(\tau) = \sum_{i=1}^{\infty} b_i \exp(-\lambda_i |\tau|) \ , \tag{1.1}$$

where b_i, λ_i – are known parameters whose physical meanings will be explained later on. Such kind of processes might be modeled as solutions of *N*-dimensional diffusive- isotropic SDE (see appendix A2 and [18] for details).

2. If the PDF of *x*(*t*) is a priori known or assumed, *W*(*x*), then the synthesis of the SDE can be done analytically (see subsection II.2).

3. Several approximations (not limited to those mentioned below) can be applied in order to assume a priori information in the form of *W*(*x*):

- Gaussian approximation when *W*(*x*) is considered as *N*-dimensional Gaussian PDF. Note that for the models based on "birth-and-death" processes, the covariation matrix for *W*(*x*) is three-diagonal (see appendix A1) and correspond to (1.1).
- Functional approximation in the form (see, for example, [19]):

$$W(\boldsymbol{x}) = \prod_{j=1}^{N} W(x_j)\left[1 + \sum_{n=2}^{N}\sum_{m=1}^{n-1}\frac{R_{nm}}{R_{mn}R_{mm}}(x_n - <x_n>)(x_m - <x_m>)\right] \qquad (1.2)$$

where it can be assumed that all the $W(x_j)$ are one-dimensional Gaussian PDF's with the first two moments obtained through the set of N independent "birth-and-death" models, the matrix $[R_{nm}]_{n-m}^{N}$ is a three-diagonal one (see details at appendix A3) and $<x_n>$ - is the mean value for the *each* component.

Once more, assuming that for large intensities of state changes the distributions for $W(x_j)$ asymptotically tend to Gaussian PDF's (see also section III), their means and variances might be found from their "prelimited" models such as "birth-and-death" models.

In this regard one has to notice that direct evaluation of those means and variances for stationary conditions for the birth-and-death models does not exist so far [15], at least to the best of our knowledge.

Though, the appendix A3 presents an approximate approach for evaluation of the unknown means and variances based on the modeling of "birth-and-death" random processes as an assembling of two statistically independent binary random processes.

So, concluding the above proposal, it is reasonable to shift the N-dimensional B-RAN modeling from N-dimensional continuous time Markov processes with discrete state space to the N-dimensional continuous time Markov processes with continuous state space for the extreme case of $N>>1$ which opens the possibility of obtaining analytical solutions. For discrete state space models analytical solutions are hardly possible.
Next, for B-RAN modeling, it might be reasonable to get analytical solutions in two extremes $N=1$ and $N>>1$ in order to get its "dynamics" for different scenarios to evaluate, for example, the service latency, etc.

2.1 Synthesis of the birth-and-death model (one-dimensional case)

Please note, that in the following the term "synthesis" of certain models means, the procedure of finding the "content" of the model, applying a priori stochastic information: stationary N-dimensional PDF and covariance function of the B-RAN process (for non-linear models), only covariance function, etc.

The expression "content" of the model means the SDE components (for diffusive-isotropic Markov models), components of the matrix A for the birth-and-death model, etc.

Concretely, the material of this subsection is devoted to the synthesis of the matrix A, applying the a priori known covariance function $B_x(\tau)$ in the form (1.1) and is based on the material of appendix A1. The following material can be also considered as a "methodology" for the synthesis of the "birth-and-death" one dimensional models.

In this regard it is reasonable to rewrite the three-diagonal matrix (A.1.2) (see appendix A1) in the more general way:

$$A = \begin{pmatrix} -a_1 & a_1 & .... & 0 \\ c_1 & -c_1 - a_2a_2 & & \\ 0 & c_2 & -c_2 - a_3a_3... & 0 \\ & & \ddots & \\ & & & -c_{N-1} \end{pmatrix}, \qquad (2.1.1)$$

where $c_k, a_k > 0$ and are subject to the synthesis procedure.
From the equation (see Appendix A1):

$$(\boldsymbol{l}_k, \boldsymbol{v}_j) = 0, \qquad (2.1.2)$$

one can find coordinates of the $\boldsymbol{v}_j$ vector. Then substituting at (A.1.7) $\boldsymbol{v}_j$ and $\boldsymbol{l}_k$ one can obtain (N-1) equations for a_1, ..., a_{N-1}; c_1, ..., c_{N-1}. Another (N-1) equations follow from the Viett theorem for the determinant $\left| A_j - \lambda E \right|$, whose $\{\lambda_i\}_1^N$ are a priori known from (1.1). Though, one has ($2N$-2) equations for ($2N$-2) unknown variables $\{a_k, c_k\}$ as $\lambda_1 = 0$ (by definition, see (A.1.3)). Therefore a unique solution for the problem is formally obtained!

It is worth mentioning here that for $N \geq 3$ analytical solutions for the above algorithm are hard to obtain, so the numerical approach has to be applied. But for $N = 2$ the following analytical example might be useful also for illustrative purposes.

Example:

Let the a priori information be:

$B_x = b_2 \exp\left(\lambda_2 |\tau|\right)$, $\boldsymbol{x} = (x_1, x_2)$ with stationary probabilities p_1 and p_2, and b_2 is the variance. Then $\boldsymbol{v}_1 = \boldsymbol{P} = (p_1, p_2)$ and $b_2 = x_2(x_1 - x_2)$ from (A.1.4) and it follows:

$$A = \begin{bmatrix} -a_1 & a_1 \\ a_2 & -a_2 \end{bmatrix}, \text{ which has to be synthesized.}$$

As $\lambda_2 = l$ ($\lambda_1 = 0$), the λ is nothing else but $1/\tau_{cor}$, where τ_{cor} is the covariation interval and from $A^T P = 0$ one can easily find $p_1 = \dfrac{a_1}{a_1 + a_2}$; $p_2 = \dfrac{a_2}{a_1 + a_2}$, where p_1 and p_2, are known. From the matrix A it follows:

$$\begin{vmatrix} -a_1 - \gamma & a_1 \\ a_2 & -a_2 - \gamma \end{vmatrix} = 0,$$

and $\gamma = \gamma_1$, $\gamma_2 = -(a_1 + a_2)$, then from (A.1.6) $D_1(\lambda) = -a_1 - \gamma$.

Though:

$$\boldsymbol{l}_1 = (1,1), \ \boldsymbol{l}_2 = \left(1, \frac{D_1(\lambda)}{a_1}\right) = \frac{1}{a_1}(a_1 - a_2) \ .$$

From the equation $(\boldsymbol{l}_k, \boldsymbol{v}_j) = 0$ it follows:

$$(\boldsymbol{l}_1, \boldsymbol{v}_1) = 1, \ (\boldsymbol{l}_2, \boldsymbol{v}_2) = 1$$

$$(\boldsymbol{l}_2, \boldsymbol{v}_1) = 0, \ (\boldsymbol{l}_1, \boldsymbol{v}_2) = 0.$$

So, from (2.1.2) one can get

$$\begin{cases} v_{11} + v_{21} = 1 \\ v_{11} - \frac{a_2}{a_1} v_{21} = 0 \\ v_{12} + v_{22} = 0 \\ v_{11} - \frac{a_2}{a_1} v_{22} = 1 \end{cases}$$

And

$$\boldsymbol{v}_1 = (P_1, P_2);\ \boldsymbol{v}_2 = \left(\frac{a_1}{a_1 + a_2}, -\frac{a_1}{a_1 + a_2}\right).$$

The system of equations, obtained from (A.1.4) is

$$\begin{cases} x_2 a_2^2 + x_1^2 a_1^2 = \dfrac{b_2 \lambda}{x_2 - x_1} \\ a_1 + a_2 = -\lambda \end{cases}$$

Though:

$$a_1 = \frac{\sqrt{x_1 x_2 + b_2} - x_2}{x_2 - x_1}\lambda$$

$$a_2 = \frac{x_1 - \sqrt{x_1 x_2 + b_2}}{x_2 - x_1}\lambda.$$

Finally, the matrix A has been synthesized; and from A and (A.1.2) one can find the values for the intensities μ_i and ε_i for the birth-and-death model.

This example illustrates how from a priori known covariation function $B_2(\tau)$ and stationary probabilities for a known number of states for the B-RAN process, the synthesis for the birth-and-death model can be completed. The next subsection II.2 is dedicated to the same procedure but for the *N*-dimensional SDE.

2.2 Synthesis of the B-RAN Markov models, based on the diffusive-isotropic SDE

The main goal of this subsection is to provide an algorithm for synthesis of continuous diffusive *N*-dimensional Markov models for B-RAN, based on SDE (A.2.1). In other words it is a methodology of the synthesis of the *N*-dimensional Markov model of B-RAN. As it was mentioned in the appendix A2, such processes are characterized by certain restrictions on the $B_x(\tau)$ for the process $x(t)$, i.e. their covariation function is assumed to be a monotonic function of "τ" in stable-state conditions. Assuming that for each process $X_j(i_0, i_1, ..., i_{N-1}, \sum\limits_{n=N}^{\infty} i_n)$ it is exactly the case and if $W(x)$ is a priori known, it forms the set of a priori data for synthesis (A.2.1).

One can easily find (A.2.3) that for the *i*th-component the vector function $f(x)$ is:

$$f_i(\boldsymbol{x}) = \frac{d_{ii}}{2}\frac{\partial}{\partial x_i}\ln W(\boldsymbol{x}) \tag{2.2.1}$$

Though, from (2.2.1) it directly follows the general dynamic structure for the Markov model for the B-RAN. But the matrix for $[d_{ii}]_{i=k}^{N}$ is still unknown (i.e. the $\{d_{ii}\}_i^{N}$ are unknown).

The approximate way to find them theoretically is well known (see [20,21] and the references therein):

- Apply statistical linearization for the non-linear functions (2.2.1), then
- Coefficients $\{d_{ii}\}$ can be found by applying the Doob's second theorem (see [20]) or the MatLab's System Identification Toolbox (SIT)). Note, that statistical linearization can be done with SIT as well!

It is worth stressing here that statistical linearization for the synthesis of the non-linear dynamic system (A.2.1.) is applied only for finding $\{d_{ii}\}^N$, i.e. it does not change the synthesis problem of the non-linear system by the linear one.

If the $W(\boldsymbol{x})$ is approximated by a Gaussian PDF then the system (2.2.1) is a priori linear and the synthesis procedure can be completed as it is shown in the following.

For the Gaussian approximation of $W(\boldsymbol{x})$, the theory of SDE is thoroughly treated in [20, 21].

Particularly, the SDE (2.2.1) for the linear case is:

$$\dot{x}_i = -\sum_{k=1}^{N} \alpha_{ik} x_k + n_i(t) \tag{2.2.2}$$

and $a_i(\boldsymbol{x}) = -\sum_{k=1}^{N} \alpha_{ik} x_k$, $d_{ii} = \dfrac{D_i}{2\alpha_{ii}}$, $\qquad$ (2.2.3)

where D_i is the variance for the each $x_i(t)$ component. The unknown coefficients $\{\square_{ik}\}$ can be found from the numerical solution of the system:

$$\sum_{i=1}^{N} \alpha_{ki} m_i = 0 \ , \ k = \overline{1,n}$$

where $\{m_i\}_1^N$ – is a vector of mean values for $W(\boldsymbol{x})$.

If a functional approximation for $W(\boldsymbol{x})$, (1.2), is applied with Gaussian partial PDF's for the $W(x_j)$ with parameters m_j and D_j , then each SDE for $x_i(t)$ is one-dimensional:

$$\dot{x}_i = -\alpha_i x_i + n_i(t) \ , \ i = \overline{1,N} \tag{2.2.4}$$

Then $\alpha_i = \dfrac{1}{\tau_{cor_i}}$ and $d_{ii} = \dfrac{D_i}{2\alpha_i}$, where τ_{cor_i} is the correlation interval for each $x_i(t)$.

Though, the dynamic *N*-dimensional models for the B-RAN are synthesized by "generating" *N*-dimensional SDE's (1.2.1) which in the general case are nonlinear, but for the specific Gaussian case are linear.

3 Statistical Properties of the Self-similar Traffic Models

3.1 General Approach

As it was pointed out (see the Appendix A4) the statistical behavior of the self-similar processes is characterized by a "long term dependence" which is represented by "heavy tails" in the covariation functions and PDF's. From the application point of view it is the main feature for short-scale B-RAN traffic modeling and is described exhaustively in [13-17,22]. In the following it is reasonable to invoke the results of B. Tsybakov (see for example [5,14]).

For sure the approach [5,14] has to be adjusted precisely to the "service" model of B-RAN (considering the B-RAN scenario as it was done, for example, at [3]), but that might be done later on.

An important theorem was presented in [5, 14], which proofs that for the asymptotic conditions of a large number of users, the self-similar conditions of the on/off Pareto model can be applied considering the following comment: In the scenario considered in [5,7,14] it is assumed a decomposition of the process $x(t)$ (see above),

into a sum of independent and identically distributed input flows, which is a unique coincidence (!) with the four stages B-RAN service processing presented above (see [3]).

Though, it seems that the applied on/off Pareto model might be adequate for B-RAN at least when the waiting to be included into a block, waiting for confirmation and waiting for service average times are much less than the service average time. One can assume that those conditions might be recurrent in scenarios for 5G, 6G and beyond.

That is why the on/off Pareto model might be practical for those scenarios at least as a first approximation for small-scale traffic.

Though, applying in the following the on/off Pareto model for "on" and "off" holding times, then the "length" distribution of the B-RAN service period or its traffic ratio for $N \rightarrow \infty$ and α_{on}, $\alpha_{off} < 2$ resembles a Gaussian self-similar process with $H = \dfrac{3 - \alpha_{off}}{2}$, where $\alpha_{off} < \alpha_{on}$.

The mean and variance of the Gaussian distribution were obtained in [7] and the simplified expressions are[1]:

$$\mu = \frac{\frac{\alpha_{on}}{\alpha_{on} - 1}}{\frac{\alpha_{on}}{\alpha_{on} - 1} + \frac{\alpha_{off}}{\alpha_{off} - 1}}$$

$$\sigma^2 = \frac{2T_{on}(2H^2 - 5H + 3)}{\left[2H(T_{on} + 1) - 3 - 2T_{on}\right]^2}$$

$$T_{on} = \frac{\alpha_{on}}{\alpha_{on} - 1}$$

(3.1)

So, if B-RAN has a Pareto distribution for the service periods (on, off) in the form:

$$P(\tau = l) = C_0 l^{-\alpha - 1} \ ,$$

(3.2)

where $1 < \alpha < 2$, $C_0 = \left(\sum_{l=1}^{\infty} l^{-\alpha - 1} \right)^{-1}$, then, when $N \rightarrow \infty$, it tends to Gaussian distribution with mean μ and

variance σ^2, i.e. it shows the same trend, as it was shown above from the corresponding continuous Markov "service" models which significantly simplify the application of the proposed models for practical scenarios.

3.2 Model comparison

The advantage of the application of Gaussian statistics for different trends of practical modeling is that it provides with the convenient way of their comparison for the future application through the already established "information" measures of their differences.

For example, for Gaussian distributions their "differences" depend only on the "distances" between the vectors of their means and the covariation matrixes.

Though, applying the generally defined "distances" between PDF's such as the Kullback-Leibler distance or Bhattacharya distance, one can evaluate the influence of the models on the behavior of concrete network parameters such as, for example, traffic latency, service latency, etc. Note, that this might be a special subject for research.

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[1] *Simplifications here denote that in (3.1) all parameters of the individual traffic are assumed to be equal to 1.*

It is well known that for two Gaussian distributions with equal mean vectors, the Kullback-Leibler measure generally defined as:

$$I_{1,2}(\boldsymbol{x}) = \int\limits_{R^N} W_1(\boldsymbol{x}) \ln \frac{W_1(\boldsymbol{x})}{W_2(\boldsymbol{x})} d\boldsymbol{x} \; ,$$

takes the form:

$$I_{1,2}(\boldsymbol{x}) = \frac{1}{2} \ln \frac{D_1}{D_2} \; , \tag{3.2.1}$$

where D_α, $\alpha=1,2$, are determinants of the covariation matrixes of two Gaussian PDF's $W_1(\boldsymbol{x})$ and $W_2(\boldsymbol{x})$. Comparing $I_{1,2}(\boldsymbol{x})$ separately with ε-entropy $I_\alpha(\boldsymbol{x})$, for $W_\alpha(\boldsymbol{x})$:

$$I_\alpha(\boldsymbol{x}) = \int\limits_{R^N} W_\alpha(\boldsymbol{x}) \ln W_\alpha(\boldsymbol{x}) d\boldsymbol{x} \, , \, \alpha = 1,2 \; .$$

One can formulate the "fidelity" measure for the practical modeling of both the "service" modeling for B-RAN and the user traffic modeling.

Let us take a simple example: If $I_{1,2}(\boldsymbol{x}) \ll I_\alpha(\boldsymbol{x})$, i.e. max $I_{1,2}(\boldsymbol{x}) \le \beta\%$, for β≪1. What difference between traffic latency μ from (3.1.1), calculated for $W_\alpha(\boldsymbol{x})$, $\alpha=1,2$ is practical for specific applications?

Though, what is the "degree of freedom" for selecting the parameters α_{on}, α_{off} for the model, etc?.

It is obvious that the answer can be easily found: when β≪1 then the precision of α_{on}, α_{off} might be of "β" value, as well.

4 Conclusions

The present material is dedicated to two aspects of B-RAN modeling: service modeling and user traffic modeling for the case of a large number of users and it is related to the large dimension of the equations for their correspondent mathematical modeling (see, for example the equations in section IV in [3], which belong to the $N+1$ dimensional state space).

Though, it was proposed to shift the description of the service modeling from N-dimensional continuous time and discrete space Markov models to the N-dimensional continuous time and continuous space diffusive Markov models for the case $N \gg 1$. It was shown that with these assumptions it is possible to provide a constructive way of the synthesis process for such kind of models with pre-defined a priori information of the N-dimensional PDF of the statistical properties of the states of the model.

As a practical way to obtain the mentioned PDF two approximations were assumed:

Gaussian and Functional one with the application of the birth-and-death models for calculation of the means and variances for partial Gaussian PDF's and its covariation matrix.

For the user traffic models the application of the Gaussian self-similar processes were proposed at [5,7,14] for $N \gg 1$ with the parameters calculated from prelimited ON-OFF Pareto models.

Taking into account the theorems from [5,14], the latter can be considered as an attempt for model design for the case of "mutual" modeling for the cases of both the system and the traffic features of B-RAN in a rather constructive fashion.

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Appendix

Appendix A1: General. Some properties of the Kolmogorov-Chapman equations for the birth-and-death Markov processes with continuous time

In the general case, the Kolmogorov-Chapman equation for the probabilistic description of the Markov time-homogeneous process with continuous time and discrete state is:

$$\frac{d\boldsymbol{P}(t)}{dt} = \boldsymbol{P}(t)\boldsymbol{A}, \tag{A.1.1}$$

where $\boldsymbol{A}$ – is a matrix filled by intensities of Poisson commutations of the states; P(t) – is a matrix line of unconditional probabilities of the states of the Markov process $\boldsymbol{x} = (x_1, x_2, x_3 ..., x_N)$:

$$\lim_{t \to \infty} \boldsymbol{P}(t) = \boldsymbol{P} \; ; \boldsymbol{P} = (p_1, \, ..., p_N) \; .$$

The matrix $\boldsymbol{A}$ for the birth-and-death processes $\theta(t)$ [15, 23] can be presented as a Jacobian matrix of the "three-diagonal" type in the form:

$$\boldsymbol{A} = [\alpha_{ik}], \text{ where } \alpha_{ik} = \begin{cases} -(\varepsilon_i + \mu_i), & i = k \\ -\varepsilon_i, & k = i+1 \\ \mu_i, & k = i-1 \\ 0; & \text{for other } ik \end{cases} \tag{A.1.2}$$

The matrix (A.1.2) is often known as "continuants" [15].

If the birth-and-death process $\theta(t)$ might be generally characterized in such a way that one of the states "j" of the process $\theta(t)$ with states $j = 0, 1, 2, \ldots$ in a random moment "t" can change at ± 1: $j \underset{\varepsilon_j}{\to} j+1, \; j \underset{\mu_j}{\to} j-1$;

where ε_j and μ_j are intensities of the correspondent Poisson processes, then the components of the matrix $\boldsymbol{A}$ depend on ε_j and μ_j, see (A.1.2).

Though, synthesis of the matrix $\boldsymbol{A}$ is equivalent to reconstruction of the birth-and-death process $\theta(t)$.

It is well known [24] that the normal Jacobian Matrix $\boldsymbol{A}$ has real negative eigenvalues (discrete spectrum): $\lambda_1 > \lambda_2 > \ldots > \lambda_N$, $\lambda_1 = 0$. (A.1.3)

Then the auxiliary variables for the covariation function of $\boldsymbol{x}(t)$ are [23]:

$$b_i = \left(\sum_{j=1}^N x_j p_j l_{ij}\right)\left(\sum_{k=1}^N x_j v_{ij}\right), \tag{A.1.4}$$

where $\boldsymbol{l}_k = (l_{1k}, \ldots, l_{Nk})$, $\boldsymbol{v}_k = (v_{1k}, \ldots, v_{Nk})$ – right and left eigenvectors and correspond to each λ_k, $k = \overline{1, N}$.

Then, the covariation function for the process $\boldsymbol{x}(t)$ is a monotonic function of τ [23]:

$$B(\tau) = \sum_{i=1}^N b_i \exp(\lambda_i |\tau|) \tag{A.1.5}$$

It is worth mentioning that if $\{\alpha_i\}^N$ are real and negative, (A.1.5) coincides with the covariation function of the *N*-dimensional diffusive-isotropic SDE (see A.1.2) which backs up the above stated proposal for the *N*-dimensional diffusive Markov model for the B-RAN (see also A.1.2).

Here it is reasonable to point out as well that the vector $\boldsymbol{P} = [p_1,...,p_N]^T \cong \boldsymbol{v}_i$ as $(\boldsymbol{l}_j, \boldsymbol{v}_k) = \delta_{ik}$ and $\boldsymbol{l}_1 = \underbrace{(1,...,1)}_{N}$ as

$\lambda_1 = 0$.

The determinant [24] applied in the following is

$$D_j(\boldsymbol{\lambda}) = \det\left|A_j - \boldsymbol{\lambda}E\right| , \tag{A.1.6}$$

where $\boldsymbol{E}$ – is a unit matrix and $D_0(\lambda)=1$.

Then the eigenvector's components can be defined as:

$$\boldsymbol{l}_{ji} = (-1)^{j-1} \frac{D_{j-1}(\lambda_i)}{a_1,...,a_{j-1}} , \tag{A.1.6}$$

through

$$(\boldsymbol{l}_k, \boldsymbol{v}_j) = 0 , \tag{A.1.7}$$

and it is possible to find the coordinates *v* (see the main text).

Appendix A2: General properties of the *N*-dimensional diffusive-isotropic SDE

The *N*-dimensional diffusive-isotropic SDE "generates" a special case of the *N*-dimensional diffusive Markov process of the type [18]:

$$\dot{\boldsymbol{x}}_i = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{D}\boldsymbol{\xi}(t) , \tag{A.2.1}$$

where $\boldsymbol{\xi}(t)$ is an *N*-dimensional vector of white Gaussian noises with unitary intensities, $\boldsymbol{D}$ – diagonal matrix of those intensities:

$$\boldsymbol{D} = \begin{bmatrix} d_{11} & & \\ & \ddots & \\ & & d_{NN} \end{bmatrix} .$$

The $\boldsymbol{f}(\boldsymbol{x})$ is a vector function of $\boldsymbol{x}$, which can be represented in the following way:

$$\begin{bmatrix} f_1(\boldsymbol{x}) = a_1(\boldsymbol{x}) \\ \vdots \\ f_N(\boldsymbol{x}) = a_N(\boldsymbol{x})) \end{bmatrix} .$$

Actually, it is a vector function of the "shift coefficients" for the SDE (A.2.1), both in the Stratonovich and Ito sense [18].

If the SDE (A.2.1) satisfies the so-called "potential" conditions, i.e.

$$\frac{\partial}{\partial x_j}\left[\frac{a_i(\boldsymbol{x})}{d_{ii}}\right] = \frac{\partial}{\partial x_i}\left[\frac{a_j(\boldsymbol{x})}{d_{jj}}\right] , \tag{A.2.2}$$

then the SDE (A.2.1) is called a diffusive-isotropic SDE [18] and the stable state PDF $W_{ST}(\boldsymbol{x})$ of the solution of the SDE (A.2.1) can be represented in the form of a contour integral:

$$W_{ST}(x) = C \exp\left\{2\int_{\bar{a}}^{\bar{x}}\sum_{i=1}^{N}\frac{a_i(\boldsymbol{x})}{d_{ii}}d\boldsymbol{x}\right\} , \tag{A.2.3}$$

where $\boldsymbol{a}$ is an arbitrary initial point of the contour and C is a normalization constant. As it is well known (see for example [18]), the time dependent PDF for the Markov diffusive process – $W(\boldsymbol{x},t)$ can be found as a solution of the so called Fokker-Plank-Kolmogorov (FPK) equation in partial derivatives of the type:

$$\frac{\partial W(\boldsymbol{x},t)}{\partial t} = -\sum_{i=1}^{n}\left\{\frac{\partial}{\partial x_i}\left[a_i(\boldsymbol{x})W(\boldsymbol{x},t)\right] + \frac{1}{2}\frac{\partial^2}{\partial x_i^2}\left[d_{ii}W(\boldsymbol{x},t)\right]\right\} , \tag{A.2.4}$$

where $W(\boldsymbol{x},t_0) = \delta(\boldsymbol{x},-x_0)$ – initial conditions.

 The FPK operator for (A.2.4) for the diffusive-isotropic process has several important properties:

- It is self-adjoint and symmetrical.
- Its eigenvalues are real, have discrete spectrum and obey the condition (inequalities) (A.1.3).
- The covariation function of $\boldsymbol{x}(t)$ follows (A.1.5) and its power spectrum for each component occupies the frequencies of the semi-axis $\omega \in [0,\infty)$.

So, one can see that from the point of view of its fundamental properties, both stochastic processes: N-dimensional "birth-and-death" Markov processes and N-dimensional diffusive-isotropic Markov processes are similar.

 In this sense the Kolmogorov-Chapman equation, (A.1.1), and FPK equation (A.2.4), are "equivalent", besides they are related to different types of Markov processes.

This is a possible reason to substitute the B-RAN service model as N-dimensional continuous time homogeneous Markov process with the discrete state space by N-dimensional diffusive-isotropic continuous time Markov model with continuous state space.

In the main text (subsection 2.2) it is shown that from the a priori data, SDE (A.2.1), the model synthesis is rather simple.

Appendix A3: Approximate approach for the calculation of the mean and variance for the one-dimensional "birth-and-death" model

 Consider the one-dimensional "birth-and-death" process model. Without any loss of generality it is possible to assume the model of three states $\theta(t)=(1, 0, -1)$ with intensities for state changes μ_i and ε_i $i=1,2$ [15], with the following approximate assembling through two independent stochastic binary signals $\theta_1=(1,0)$ and $\theta_2=(0,-1)$,

with parameters $0\underset{\varepsilon_1}{\to}1$; $1\underset{\mu_1}{\to}0$; $0\overset{\varepsilon_2}{\to}-1$; $-1\overset{\mu_2}{\to}0$.

One has to notice that the absence of the stationary values for the mean and variance of the birth-and-death model [15], mentioned above, can be assumed as a "product" of the time "overlapping" of the states for independent binary "components" of the model which can be "neglected" at least as a first approximation when $\forall \varepsilon_i$, $\forall \mu_i \gg 1$, i.e. when intensities of state changes are very high.

This is a "physical" reason for the approximate assumption of the existence of stationary values for the mean and variance for the proposed model, when rigorously it does not take place!

Then, for each of those binary random signals, the following equalities can be written [15]:

$$
\left.
\begin{aligned}
m_{\theta_1} &= \frac{\mu_1 - \varepsilon_1}{\varepsilon_1 + \mu_1}; \qquad D_{\theta_1} = \frac{4\varepsilon_1\mu_1}{(\mu_1 + \varepsilon_1)^2} \\
m_{\theta_2} &= \frac{\mu_2 - \varepsilon_2}{\varepsilon_2 + \mu_2}; \qquad D_{\theta_1} = \frac{4\varepsilon_2\mu_2}{(\mu_2 + \varepsilon_2)^2}
\end{aligned}
\right\}
\qquad (A.3.1)
$$

Then, obviously, for statistically independent $\theta_1(t)$ and $\theta_2(t)$, the parameters for the birth-and-death process, $\theta(t)$, are:

$$
\begin{aligned}
m_\theta &= m_{\theta_1} + m_{\theta_2} \\
D_\theta &= D_{\theta_1} + D_{\theta_1} \\
\mu &= \mu_1; \quad \varepsilon = \varepsilon_1
\end{aligned}
\qquad (A.3.2)
$$

Formally μ_2 and ε_1 can be chosen in an arbitrary way and are positive, i.e. $\varepsilon_1, \mu_2 > 0$, but for applications it is convenient to assume each binary process as a symmetrical one, i.e. $\varepsilon_1 = \mu_1$, $\varepsilon_2 = \mu_2$.

Then, it follows, that the mean $m_\square = 0$ and the variance $D_\theta = 2$. For the general case of Markov process instead of (1, 0, -1), for the "j" state it has to be considered (j+1, j, j-1), and though $D_\theta = 2j^2$ and $m_\theta = 0$, which is almost trivial for the case of symmetrical models.

Appendix A4: Basic features of the fractal self-similar and chaotic processes

A.4.1 Introductory comments

Fractal stochastic processes, self-similar stochastic processes and chaos processes have some common features that can unite them from the point of view of traffic modeling, besides that, strictly speaking, they are not similar. But their statistical features follow the so-called "replication" in time: i.e. they are "similar to itself". In other words, the dynamic systems which can be considered as "generators" of such kind of processes are somehow "deterministic", but being very "complex" in time they can be described in a stochastic way. Long time ago B. Mandelbrot illustrated that the traffic in communication networks shows the stochastic self-similar properties which will be described in the following.

A.4.2 Definition of the self-similarity and the Hurst exponent

The self-similar process is defined as a real process $X(t)$ with the Hurst exponent H >0 if for any a>0 the finite-dimensional PDF for $X(at)$ a>0 is identical to the finite-dimensional PDF for $a^H X(t)$. So, it can be supposed that for self-similar processes, the plots are "identical" in the time scale and its statistical characterization does not "change" for this time scale. The Hurst exponent "indicates" (see above) for how long this spectacular property is preserved (it is called "long-range" dependence), which obviously diminishes as H grows. So, this long-range dependence feature defined by H (actually $0.5 \le H \le 2$) provides the "long-tail" covariation functions and the long (heavy) tail distributions (PDF's). Practically for long-range dependence the most important values are $H = 0.5; 1$. Examples of heavy tailed PDF's (applied for self-similar traffic models) such as Pareto, logarithmic gamma distribution, logarithmic Earlang distribution, etc. can be found at the preprint [6] and in [14].

More detailed, but more complex, traffic modeling with the so-called fractional Levy Motion tends to the $\square$-stable distributions (see [14]); α is the fractal space dimension, $0.5 \le \alpha \delta 2$ ($\alpha = 2$ means the Gaussian distribution). Those models are mathematically described by dynamic systems, particularly of Hamiltonian type, which show both fractal and chaotic properties (see [19] and references therein). These chaotic dynamic systems with fractal kinetics are usually named as "strange attractors" and can be described, for its statistical

characterization, with fractional Fokker-Plank equation, FFPE, (see [19] and the references therein). In some sense FFPE and FPK (see [19]) are analogous but the first one, contrary to FPK, depends on the so-called "fractal space".

One has to notice that the analogy between FFPE and FPK does not mean that it is possible to find similar methods for synthesis of the dynamic systems with the mixed fractal and chaotic properties.

It is not the case, and the PDF of the α−stable distributions (see [19, 22)]) are hardly possible to apply at least for analytical studies (see [14, 22]), except on three special cases $\alpha = 0.5; 1; 2$. Details can be found at [22]. One has to notice that the special case $\alpha = 2$ (Gaussian) can be modeled as it was commented in appendix A2.