British Journal of Applied Science & Technology
3(1): 34-47, 2013

SCIENCEDOMAIN *international*
www.sciencedomain.org

Determination of Normal Operating Heights of Reservoirs in a Network Using Non-linear Reservoir Method

M. K. A. Abdulrahman[1,2*], A. A. Moh'd Ismail[1] and Sie Long Kek[3]

[1]*Department of Mathematics, Universiti Teknologi Malaysia, Malaysia.*
[2]*Department of Maths., Stats., and Computer Science, Kaduna Polytechnic, Nigeria.*
[3]*Department of Mathematics, Universiti Tun Hussein Onn, Malaysia.*

Authors' contributions

This work was carried out in collaboration between all authors. Author MKAA designed the study, performed the model formulation and result analysis, and wrote the first draft of the manuscript. Author AAMI supervised, provided literatures and managed the entire study. Author SLK coordinated and revised the manuscript. All authors read and approved the final manuscript.

Research Article

Received 19th July 2012
Accepted 21st November 2012
Published 17th December 2012

ABSTRACT

In reservoir modeling, making the steady-state policy of the flood control is a challenging task. In this paper, the normal operating heights of a reservoir network is discussed. Initially, the coupled tank system is employed to model the reservoir network. At the equilibrium state, the normal operating heights are then derived analytically. As the linearization is taken, the associated Jacobian matrix, which is a diagonally dominant tridiagonal matrix with negative diagonal entries, is obtained. Because of the negative eigenvalues of this matrix, the stability condition of the reservoir network can be made. For illustration, the different rain rates are used to calculate the surface area of the reservoir and the corresponding normal operating heights of the reservoir are then determined. The result obtained shows the stability of the reservoir. In conclusion, the approach discussed is efficient to the decision making on the stability of a reservoir network.

Keywords: Reservoir network; coupled tank system; tridiagonal matrix; flood control; steady state.

**Corresponding author: E-mail: kaabdulrahman2@live.utm.my;*

1. INTRODUCTION

Flood phenomena are the major environmental issue that is confronted by both rural and urban settlements on a global scale. In this point of view, a sustainable approach for flood management needs to be developed. However, because of the factors, such as hydrology, hydraulics, topology, vegetation, perviousness and environmental standards [1], the modeling of the storage of water in a reservoir and the water transfer between two or more reservoirs become complex.

To overcome these complexities, the routing method, which is an analytical procedure, is used to evaluate the water flow in hydrological systems by taking the rainfall runoff as the input to the reservoir. The hydrological routing method utilizes the continuity equation to derive the Muskingum equation [2], which is known as the channel routing approach since the reservoir activities are taking place along the conveyance. Muskingum-Cunge method was proposed to evaluate parameters for Muskingum method based on hydraulic characteristics of the channel [2]. This approach is applied as a base model for hydrologic drainage network channel flow routing model to evaluate drainage network topology, hydraulic properties of cross section, routing the flow in individual channel and in the network [2].

Table 1. Variables and parameters

Parameters	Description
A_j	Surface area of j^{th} reservoir (m^2)
A_{scj}	Area of sub-catchment (m^2)
a_j	Size of orifice (m^2)
$c_{d,j}$	Discharge coefficient of R_j
f	Average infiltration rate (m^3/s)
g	Acceleration due to gravity (m/s^2)
H_j	Head loss in reservoir $R_j\,(m)$
r_j	Radius of orifice of $R_j\,(m)$
h_j^{perm}	Permanent height of the retention pond (m)
h_j^{temp}	Temporary height of the retention pond (m)
h_j^{flood}	Flood height of the retention pond (m)
H_j^0	Normal operating height $R_j\,(m)$
I	Rainfall intensity or inflow (m^3/s)
$Q_{o,j}$	outflow or discharges from reservoir *Rj* (m^3/s)
t_c	Time of concentration (s)
L	Length of the weir (m)
V_j	Volume of reservoir *j* ***(m³)***
R_j	j^{th} Reservoir

Nonlinear reservoir routing method is used for flood control in different catchments [2,3]. The solution of nonlinear reservoir approach can be obtained by using the finite difference method [3]. In this approach, the water level in the reservoir is monitored between two fixed points using the variation in inflow and outflow of water within a time interval. Then, a single differential equation with two unknowns can be solved. Here, the inflow consists of runoff and discharges from upland reservoir, while the outflow is divided as evapotranspiration, infiltration and discharges to reservoir on the lower side. Since a single pond cannot control the flooding effectively in a catchment [4-6], it is important to consider the application of the nonlinear reservoir method for a network of reservoirs.

It is noticed that detention and retention ponds are used for flood control, where detention pond stores water temporarily, and retention pond stores water permanently. Usually, detention pond is designed to have a maximum capacity corresponding to the time of concentration for a return period of 25 years [7-9]. Nevertheless, the retention basin is a successful flood control reservoir, especially for pervious soil with the frequent storm event [5]. In this case, retention pond must have an enough capacity to store the complete runoff which is at least to its permanent mark. Retention pond has three to four basic heights, where the permanent height always has a fixed pool of water level at any time. The temporary height is water level which is expected at short duration of time in case of any prolonged rain event and the flood height is the point of extreme rain event and high rain rate [10]. The fourth height is the dead height which is designed for sediment volume. From [11], it is reported that the return period of the post-development of peak discharge rate is ranged from 2 to100 years and the corresponding pre-development level is often used as the flood control criterion. In addition, hydrological analysis of the detention pond with the associated probabilistic approach for pond sizing was extensively discussed as in [12-14]. Comparison of kinematic wave and nonlinear reservoir method was extensively explored in the work of [15]. A full description of the routing methods and the selection criteria for the specific uses are fully discussed and reported in [6].

Experimentally, the coupled tank system approach is usually applied to control the water level between tanks, where the inflow and outflow through the orifice are employed [16,17]. Based on the design consideration of the detention and retention ponds in sizing and location [5], the coupled tank system approach is then extended to the reservoir network for flood control using the nonlinear reservoir model. It is found that nonlinear reservoir model together with the couple tank systems can be used to study reservoir network for flood control. In our work, a series connection of reservoir network is considered to study the stability of the network by analytical determination of normal operating heights that can guarantee free flow of water in the reservoir network.

The solution procedure of a system of nonlinear ordinary differential equations (ODEs) is problem dependent, which is determined by the degree of its nonlinearity. To a system engineer, the linearization approach could be the best option to design the controller. In doing so, the linearized system must satisfy certain conditions, which revolve around the stability of the system. The stability analysis will give a clue if linearized system can be used to approximate the nonlinear system around the steady state [18,19,20]. Therefore, making a decision on the stability of a reservoir network is useful in the reservoir modeling. In the following sections, the nonlinearity of the proposed model formulation is critically described. The criterion of the stability and the normal operating heights are determined analytically for each reservoir in the network.

2. MODEL FORMULATION

This work relies on the coupled tank system approach to model a network of ponds for flood control. It is mentioned that a single pond would not be used here because the single pond in a catchment cannot control flood effectively. The aim of the proposed model formulation is to determine the normal operating heights of reservoirs in a network. Based on the designed parameters of the catchment under study, a free flow of water is maintained according to the criterion that will be determined later.

To establish the foundation for this work, the respective variables, constants and parameters used in this formulation are listed in Table 1. The following are assumptions which this work relied on:

1. All ponds in the network are detention ponds except the last down-most pond is the retention pond.
2. The return period for detention ponds is less than that of retention pond designed.
3. The capacity design of all detention ponds must not exceed the permanent storage of the retention pond. That is,

$$v + \sum_{j=1}^{m-1} V_j(h_j) \le V_m(h_m^{perm})$$

where v is the initial volume of water in the retention pond R_m and V_j's is the volume of water in the detention ponds R_j, *j = 1,2,…,m -1,* for a given rain event.

The volume of water in a reservoir at any instant is given by

$$V = V_{stored} + V_{flowin} + V_{precipitation} - V_{flowout} - V_{evaporation} - V_{seepage} \quad (1)$$

Equation (1) gives a full description of water balance in a reservoir, but the different form of the water balance equation can be used in the reservoir modeling. Consideration of precipitation, evaporation and seepage varies considerably between catchments, but it is less significant in most cases where activity happens over a short period of time [21]. Several routing models have been developed for a single reservoir in a network to capture this loss. One of the methods used for water routing is the nonlinear reservoir routing method, which utilizes the changes in the height of water in the reservoir with runoff as input and discharges as output. This input-output method is particularly considered as an acceptable routing technique for storm duration, which is longer than the watershed time of concentration [3,15]. From(1), the change volume of water in the reservoir can be expressed as change in the height of a reservoir. According to the conservation law, it is expressed as

$$A\frac{dH}{dt} = AI - Q_o \quad (2)$$

where the discharges are either through an orifice or a weir as given below:

$$Q_{o,j} = c_{d,j} a_j (2gH_j)^{1/2} \quad \text{or} \quad Q_{o,j} = c_{d,j} L H_j^{3/2}$$

Here, $c_{d,j}$ is the discharge coefficient of j^{th} reservoir, a_j is the cross-sectional area of orifice of j^{th} reservoir, L is the length of weir, H_j is the head loss and g is the gravitational constant.

Equation (2) can be extended to a two-connected reservoir in a network as given below:

$$\left.\begin{aligned} A_1 \frac{dH_1}{dt} &= A_1 I - Q_{0,1} \\ A_2 \frac{dH_2}{dt} &= A_2 I + Q_{o,1} - Q_{o,2} - A_2 f \end{aligned}\right\} \tag{3}$$

where the infiltration term f in the right-hand side of the second equation in (3) is due to the function of reservoir as a retention pond. Using either orifice or weir does not affect the model formulation.

For *m* reservoirs, the flood control network is

$$\left.\begin{aligned} A_j \frac{dH_j}{dt} &= A_{sc_j} I + B_i \sqrt{H_i - H_j} - B_j \sqrt{H_j - H_k} \quad ; j = 1, 2, \dots, m-1 \\ A_m \frac{dH_m}{dt} &= A_{sc_m} I + B_{m-1} \sqrt{H_{m-1} - H_m} - B_m H_m^{3/2} - A_m f \end{aligned}\right\} \tag{4}$$

Where i is the index of inflow to the reference reservoir R_j, and j is index of reference reservoir and k is the index of outflow from reference reservoir R_j, $B_j = c_{d,j} a_j \sqrt{2g}$ for $j = 1, 2, \dots, m-1$, $B_m = c_{d,m} L$ and $H_j = H_{R_j} - r_j$ where H_{R_j} is the height of water from the base of the reservoir to the surface of water.

Equation (4) is a system of nonlinear ODEs which represents the relationship for storage, inflow and outflow of a reservoir in the network. The solution procedure to this nonlinear system is generally problem dependent upon the degree of nonlinearity. To overcome this complexity, we normalize (4) about the normal operating heights of each reservoir, and followed by studying the stability of(4). In this way, the solution to the linearized system can be used as an approximate solution of the nonlinear system. Here, we define the normal operating height as the height which the water level in each reservoir is expected to maintain free flow. It is important to note that the normal operating heights are equilibrium points for which the stability of a nonlinear system can be presented. These normal operating heights can also be interpreted as the steady state heights, which are defined as

$$\begin{aligned} H_j(s) = H_j^0 = h_j^{\max} \quad &; \quad V(h_j^{\max} + r_j) = V_j^{\max} \quad ; \quad j = 1, 2, \dots, m-1 \\ H_m(s) = H_m^0 = h_m^{perm} \quad &; \quad V(h_m^{perm} + r_m) = V_m^{perm} \end{aligned}$$

where $h_j^{\max}$ is the maximum height of each detention pond, h_m^{perm} is the permanent height of retention pond, $V_j^{\max}$ is the volume for the maximum height $h_j^{\max}$, and V_m^{perm} is the volume for the permanent height h_m^{perm}. The three major designed capacities of retention pond are the volume of permanent height $V_j^{perm} = V(h_j^{perm})$, the volume of temporary height $V_j^{temp} = V(h_j^{temp})$, and the volume of flood height $V_j^{flood} = V(h_j^{flood})$. These volumes are designed for different return periods [10].

From Assumption 3, we find that at any time $t \leq t_c$, the volume of retention pond is derived from the volume of all the detention ponds [10]. That is

$$V_m(h_m^{perm}) = V_1(h_1^{\max}) + V_2(h_2^{\max}) + ... + V_{m-1}(h_{m-1}^{\max}) + v$$

This policy ensures that the capacity of the retention pond can fulfill the quantity and quality of water needed for recreation and guide against flood downstream.

The linearization of (4) about the normal operating heights is

$$\left.\begin{aligned} \frac{dh_j}{dt} &= \frac{B_i}{2A_j\sqrt{H_i^0 - H_j^0}}h_i - \left(\frac{B_i}{2A_j\sqrt{H_i^0 - H_j^0}} + \frac{B_j}{2A_j\sqrt{H_j^0 - H_k^0}}\right)h_j + \frac{B_j}{2A_j\sqrt{H_j^0 - H_k^0}}h_k \\ & j = 1, 2, \dots, m-1, \\ \frac{dh_m}{dt} &= \frac{B_{m-1}}{2A_m\sqrt{H_{m-1}^0 - H_m^0}}h_{m-1} - \left(\frac{B_{m-1}}{2A_m\sqrt{H_{m-1}^0 - H_m^0}} + \frac{3}{2}B_m(H_m^0)^{1/2}\right)h_m \end{aligned}\right\} \quad (5)$$

with $H_j = H_j^o + h_j$ and $\dot{H}_j = \dot{h}_j$, where H_j^0 is a constant.

The normal operating heights are conceptualized as the steady state which will guarantee the stability of the system given in (4). Equation (5) is a system of linear equations which gives a linearized form of the nonlinear system (4) about the steady state heights. This equation can be expressed in terms of the steady state heights by replacing the normal operating height H_j^0 with the steady state height $H_j(s)$. Linearization of a nonlinear system may not in general produce a better approximation, but in order to guarantee the linearized system behaves as well as the original nonlinear system, the linearized system must be stable about the steady state heights [22, 23]. The degree of nonlinearity of a system varies between mildly and highly nonlinearities. In practice, linearizing a nonlinear system is a common practice and the result obtained is comparable with the original nonlinear system. Bifurcation analysis shows that the system is stable about its steady state [19,24]. As the solution of (5) approximates the solution of (4), the height in each reservoir has to be bounded, which is between the minimum and maximum heights, so that a tractable solution can be obtained. The minimum point may not necessarily a zero height but is the height corresponding to zero volume. In stability study, an accurate description of the system is provided without necessarily solving the problem. Studying the stability behavior of a system is to determine the condition of free flow of water in a reservoir network. For the reservoir R_j, j =1, 2, …, m, the system matrix associated with (5) is the Jacobian matrix associated with (4) which can be used as the decision making tool on the stability of a reservoir network.

3. STABILITY ANALYSIS OF THE STEADY STATE HEIGHTS

The stability of the system (4) is studied around the steady state height which is obtained by solving (4) when $\dot{H}_j = 0 \ \forall j$. Starting from the inflow *i=0* in (4) for *j=1,2,…,m-1,* we have

$$H_j(s) = H_k(s) + \left(\frac{A_{sc_j} I}{B_j}\right)^2 \ ; \ i = 0$$

where *i =0* indicates that there is no inflow to the reservoir *j* and the reservoir *k* is the lower reservoir that takes water from the reservoir *j*.

The term of $H_j(s)$ serves as the input to all reservoirs which receives inflow from the upper reservoirs, and in general, the steady state height of *m* connected reservoirs network is

$$\left.\begin{aligned} H_j(s) &= \left(\frac{I}{B_j}\sum_{i=1}^{j} A_{sc_i}\right)^2 + H_k(s) \;\; ; \;\; j \in (1,2,...,m-1) \\ H_m(s) &= \left(\frac{1}{B_m}\left(I\sum_{i=1}^{m} A_{sc_i} - A_m f\right)\right)^{2/3} \end{aligned}\right\} \qquad (6)$$

$$i \neq 0, \;\; j := i \;\; and \;\; k := j$$

where $i \neq 0$ indicate the sets of reservoirs that receive water from upper reservoirs. For subsequent determination of normal operating heights, the formal reference reservoir *j* becomes the inflow to the next reservoir and the formal reservoir that receives water from the reference reservoir becomes the new reference reservoir, that is, $j := i$ and $k := j.$

Equation (6) shows that there is an inter-relationship between the steady state heights of two adjacent reservoirs from the first reservoir down to the m^{th} reservoir. The full description of the steady state may not have been reflected in (6) since water discharge from the m^{th} reservoir is connected to stream which also has its normal operating height. Incorporating this into (4) the network model is

$$\left.\begin{aligned} A_j \frac{dH_j}{dt} &= A_{sc_j} I + B_i\sqrt{H_i - H_j} - B_j\sqrt{H_j - H_k} \qquad ; j = 1,2,\ldots,m-1 \\ A_m \frac{dH_m}{dt} &= A_{sc_m} I + B_{m-1}\sqrt{H_{m-1} - H_m} - B_m\left(H_m - H_s\right)^{3/2} - A_m f \end{aligned}\right\} \qquad (7)$$

and similar to (6), the steady state becomes

$$\left.\begin{aligned} H_j(s) &= \left(\frac{I}{B_j}\sum_{i=1}^{j} A_{sc_i}\right)^2 + H_k(s) \;\; ; \;\; j = 1,2,...,m-1 \\ H_m(s) &= \left(\frac{1}{B_m}\left(I\sum_{i=1}^{m} A_{sc_i} - A_m f\right)\right)^{2/3} + H_s(s) \end{aligned}\right\} \qquad (8)$$

$$i \neq 0, \;\; j := i \;\; and \;\; k := j$$

Equation (8) shows that the entire normal operating height of the reservoir in the network can be estimated from the normal operating height of the connecting stream or river. Consider dropping of the discharge term in the m^{th} term of (7) and substituting (8) for H_m in (7), we have

$$I\sum_{i=1}^{m} A_{sc_i} - A_m f = 0 \qquad (9)$$

This can be simply interpreted that the total runoff in the entire catchment is equal to the total water loss due to infiltration in the retention pond. The absence of the height in (9) is an indication that there cannot be normal operating height at the m^{th} reservoir when there is no discharge from the retention pond. The steady state heights obtained from (6) can be substituted in (5) to get the coefficient matrix of the system. This matrix can also be generated by taking

$$\partial \dot{H}_j / \partial H_r ; \qquad r = i, j, k$$

Bifurcation analysis relies on the Jacobian matrix that is evaluated at the steady state height to determine the stability of the reservoir network. The lumped hydrologic and hydraulic parameters in the Jacobian matrix have influence on the nature of the matrix that will be used for stability study. The number of reservoirs in the network determines the dimension of the matrix while the network configuration determines its nature. A series connection of reservoir will produce a $m \times m$ non-symmetric tridiagonal diagonally dominant matrix, while a hypothetical network will produce a $m \times m$ non-symmetric diagonally dominant matrix. Theoretically, the unequal area of the sub-catchment is responsible for the non-symmetric nature of these matrices. Equal areas are, however, rare to come across in practice.

Let the Jacobian matrix associated with (7) be $H(s)$ and λ_j is the corresponding eigenvalues. From Poincare Lyapunouv Theorem, if $\lambda_j \neq 0$, for each *j*, the steady state is said to be hyperbolic and the trajectory of a nonlinear system around the steady state behaves in the same way as the associated linear system obtained through the linearization of this nonlinear system [23]. In fact, the Jacobian matrix is the best linear approximation to a continuously differentiable function near a given equilibrium point [25]. The decision on the nature of stability of the steady state rests on the eigenvalue or the definiteness of the matrix. In this formulation, the steady state obtained will produce a negative definite Jacobian matrix which is required to get the global stability for the system.

By similarity transformation, the non-symmetric tridiagonal matrix can be transformed into a symmetric matrix [26-28]. This transformation confirms that the eigenvalues are real and equal to the eigenvalues of the original non-symmetric matrix [26]. The condition $a_{j+1,\,j} / a_{j,\,j+1} > 0$, for each *j,* is satisfied. That is, none of the sub-diagonal element must be zero or have alternating sign. For $\lambda_j > 0$, the steady state is a source and the system is unstable, whereas for $\lambda_j < 0,$ the steady state is a sink and the system is stable. But, if λ_j admits both positive and negative signs, then the steady state is a saddle point [22]. In our study, a system in which all the eigenvalues are negative is wanted. This will guarantee the free flow of water in the catchment keeping the water level around the normal operating heights.

The following proposition is a direct consequence of Proposition (2.1) in [26] which is used to consider the negative diagonal entries in our work.

3.1 Proposition 1

Let A_n be the real symmetric tridiagonal matrix with the diagonal entries negative. If

$$b_i^2 \le \frac{1}{4} a_i a_{i+1} \left(1 + \frac{\pi^2}{1 + 4n^2} \right)$$

for *i=1,2,…,n-1,* where a_i and b_i are the tridiagonal entries of the matrix A_n, then the determinant of A_n satisfies

$$\det(A_n) < 0.$$

Proof:

$$Given \;\; A_n = \begin{pmatrix} -a_1 & b_1 & & & \\ b_1 & -a_2 & b_2 & & \\ & b_2 & \ddots & \ddots & \\ & & \ddots & \ddots & b_{n-1} \\ & & & b_{n-1} & -a_n \end{pmatrix}$$

$Let \; A_n' = -A_n$, $then \; b_i < 0$, $a_i > 0 \; and \; a_i a_{i+1} > 0$.

Since $b^2 \le \frac{1}{4} a_i a_{i+1} \left(1 + \frac{\pi^2}{1 + 4n^2} \right)$, *then* A_n' *is positive definite. That is,* $\det(A_n') > 0$.

Therefore, A_n *is negative definite since* $A_n = -A_n'$. □

Remark: Proposition 1 shows that the normal operating heights which produced the Jacobian matrix will always make the network stable.

4. EXAMPLE

Consider a catchment endowed with 10 sub-catchments, which is connected in series as shown in Figure 1, and the hydrologic and hydraulic parameters that are given in Table 2.

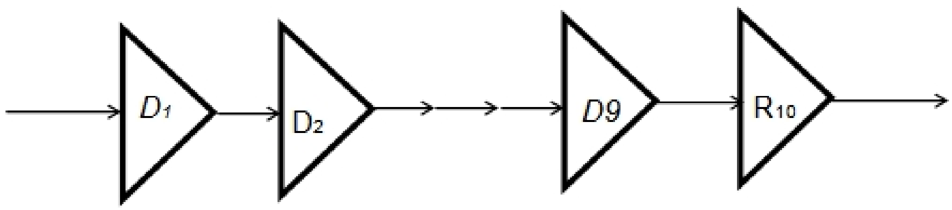

Figure 1. Network of 10 reservoirs connected in series

For each reservoir in the network, the normal operating heights that will guarantee free flow of water are determined.

5. RESULTS AND DISCUSSIONS

Figure 1 shows the series connection of network with 10 reservoirs. The associated Jacobian matrix given in (10) is generated by the coefficient of the linearized system in (5) and is evaluated at the normal operating heights or the steady state height in (6) which was obtained by setting (4) to zero. This Jacobian matrix is a non-symmetric tridiagonal matrix with the entries obtained from the hydrologic and hydraulic parameters of each sub-catchment. Notice that the non-symmetric behavior of the tridiagonal matrix is caused by the surface area in each reservoir. Nevertheless, the diagonally dominant property of the original matrix is retained and the symmetric behavior of the transformed matrix can be used to determine the definiteness. As a result, the definiteness of the original matrix is guaranteed, as given below:

$$H(s)=\begin{pmatrix} -\frac{\delta_1}{A_1} & \frac{\delta_1}{A_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\delta_1}{A_2} & -\left(\frac{\delta_1}{A_2}+\frac{\delta_2}{A_2}\right) & \frac{\delta_2}{A_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\delta_2}{A_3} & -\left(\frac{\delta_2}{A_3}+\frac{\delta_3}{A_3}\right) & \frac{\delta_3}{A_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\delta_3}{A_4} & -\left(\frac{\delta_3}{A_4}+\frac{\delta_4}{A_4}\right) & \frac{\delta_4}{A_4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\delta_4}{A_5} & -\left(\frac{\delta_4}{A_5}+\frac{\delta_5}{A_5}\right) & \frac{\delta_5}{A_5} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\delta_5}{A_6} & -\left(\frac{\delta_5}{A_6}+\frac{\delta_6}{A_6}\right) & \frac{\delta_6}{A_6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\delta_6}{A_7} & -\left(\frac{\delta_6}{A_7}+\frac{\delta_7}{A_7}\right) & \frac{\delta_7}{A_7} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\delta_7}{A_8} & -\left(\frac{\delta_7}{A_8}+\frac{\delta_8}{A_8}\right) & \frac{\delta_8}{A_8} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\delta_8}{A_9} & -\left(\frac{\delta_8}{A_9}+\frac{\delta_9}{A_9}\right) & \frac{\delta_9}{A_9} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\delta_9}{A_{10}} & -\left(\frac{\delta_9}{A_{10}}+\frac{\delta}{A_{10}}\right) \end{pmatrix} \quad (10)$$

with

$$\delta_j = \frac{B_j^2}{2I\sum_{i=1}^{j} A_{sc_i}}\ ; \quad j = 1, 2, \ldots, m-1$$

$$\delta = \frac{3}{2} B_m^{1/3}\left(I\sum_{i=1}^{m} A_{sc_i} - A_m f \right)^{2/3}$$

$$A_j = \left(\frac{A_{sc_j} I_{sc_j} t_c}{k} \right)^{1/\tau} \ ; \quad A_j < A_{sc_j}$$

where the relationship between the area of catchment and the area of reservoir was established in [29] and the value of k and τ are, respectively, 1/120 and 3/2 as discussed in [30]. These two constants are used to determine the surface area of the reservoirs which are part of the input of decision matrix.

We obtain negative eigenvalues or negative definiteness as criteria that guarantee free flow or stability of the system. It was observed in [26] that the general characterization of positive definiteness, which is given by eigenvalues, seems inadequate as far as the applied and numerical matrix theories are concerned. This is because of the enormous computational efforts that are involved in the calculations of the eigenvalues. Depending on the magnitude of the problem and availability of computational tools, the principal minors due to Sylvester's theorem, can be used as decision criteria. This is exploited in the proof of the proposition on the negative definiteness of the decision matrix.

The results obtained in Table 2 shows that the surface area of the reservoirs is determined by the rain intensities and the corresponding time of concentration. To derive the area and volume relationship of the reservoirs in a catchment, the pyramidal consideration was used in [30] to get the relationship.

Table 2. Computed normal operating heights and respective eigenvalues

j	$c_{d,j}$	A_{sc_j}	a_j	t_c	r_j	A_j			$H_j(s)$			λ_j		
						Min(I)	***Ave(I)***	***Max(I)***	***Min(I)***	***Ave(I)***	***Max(I)***	***Min(I)***	***Ave(I)***	***Max(I)***
1	0.33	2200	1.0	11.13	0.564	110.8649	338.991	594.4526	13.5728	338.076	1797.099	-0.1601	-0.0108	0.0024
2	0.32	3100	1.5	19.20	0.691	200.3954	612.7479	1074.511	13.215	327.847	1741.942	-0.0629	-0.0050	-0.0009
3	0.62	600	2.0	17.96	0.798	64.13369	196.1012	343.8819	12.234	299.789	1590.638	-0.0254	-0.0019	-0.0004
4	0.33	1850	2.5	20.67	0.892	149.1982	456.2026	799.994	12.051	294.579	1562.543	-0.0184	-0.0016	-0.0003
5	0.32	4600	2.5	19.03	0.892	259.2085	792.5805	1389.864	11.341	274.270	1453.027	-0.0003	-0.0000	-0.0000
6	0.62	1630	2.5	20.13	0.892	134.7377	411.9867	722.4573	9.422	219.423	1157.271	-0.0022	-0.0001	-0.0000
7	0.33	1780	3.0	31.88	0.978	194.1055	593.5153	1040.785	8.767	200.702	1056.315	-0.0051	-0.0004	-0.0001
8	0.32	7400	3.5	33.99	1.055	523.8722	1601.841	2808.979	6.727	142.379	741.814	-0.0109	-0.0011	-0.0002
9	0.62	8140	4.0	30.00	1.128	513.6149	1570.477	2753.979	3.285	43.970	211.148	-0.0134	-0.0008	-0.0002
10	0.33	9000	5.0	27.00	1.261	511.9364	1565.345	2744.979	2.003	7.312	13.466	-7.0146	-13.385	-18.164

g=9.8m/s, L=10m, f=0.013m3/s, k=1/120, τ=3/2, Min(I)=0.0003972m3/s Ave(I)=0.0021239m3/s, Max(I)=0.00499319m3/s

Remark: In Table 2, the surface area A for the minimum rainfall intensity Min(I), average rainfall intensity Ave(I), and maximum rainfall intensity Max(I) are calculated to determine the respective normal operating heights $H_j(s)$ and their eigenvalues λ_j. The zeros that appear in the eigenvalue column is due to approximation. The actual values are *-2.6042X10*$^{-5}$ in *Ave(I)*, *-5.0700X10*$^{-6}$ and *-3.2414X10*$^{-5}$ in *Max(I)* column respectively.

Thus, we apply this relationship to determine the surface area of the reservoirs in a catchment [29]. In such a way, the associated normal operating heights are determined for each sub-catchment in each rain rate.

In our discussed network, the normal operating heights have been evaluated by the parameters that make up (10) which Proposition 1 has shown to guarantee stability. On the other hand, for a network with data on normal operating height, (8) is substituted in each coefficient of h_j in (5) to get (10). The stability of the reservoirs is then determined by finding the principal minors or the eigenvalues of (10). There are a number of reasons that the stability test may fail, these includes prolonged rain event, high rain intensity, short inter event time and sediment accumulation. These factors are rare in occurrence for a good design since return period of 100 years are usually used for flood design. The only factor which has a great effect on the others is the sediment accumulation. It affects the effective capacity of the reservoirs and a decrease in all design parameters which eventually result in frequent flooding. It is suggested that the minimum sediment should be required, and the flow pattern and the hydraulic selection might be redesigned to restore the stability of the reservoirs.

6. CONCLUSIONS

The stability of a series connection of a reservoir network configuration was discussed in this paper. This was done by calculating the surface area and the normal operating heights of the reservoirs from the catchment parameters. The network configuration, which gives the symmetric diagonally dominant tridiagonal matrix when the similarity transformation is applied, makes the computation and the reservoir network analysis useful and interesting. Retention pond is the most important reservoir in the network because the water flow in the network can be regulated and the discharges can be controlled to the connecting stream. The effect of the connecting stream on the stability of the network has been shown in (7) and the steady state in (8) is considered as the normal operating height of the connecting stream on the stable situation. To capture the stability behavior effectively, the network is examined at the following heights:

$$\text{(a)}\ \dot{h}_j = 0;\ h_j = h_j^{\max}\ as\ \ t \to \infty;\ \ j = 1, 2, ..., m-1$$

$$\text{(b)}\ \dot{h}_m = 0;\ h_m \in (h_m^{perm}, h_m^{flood})$$

As a result, the coupled equations can allow us to determine the time for water to rise from the normal operating height h_m^{perm} and the flood height h_m^{flood} in the catchment. In addition, the time estimate for flood occurrence can be reduced as the reservoir lost its effective capacity due to sediment accumulation. In the future research, a complete model description with considering the sediment accumulation will be taken into account for the stability of the network. It is expected that a controller design can provide a foundation of the optimal control policy in reservoir modeling.

ACKNOWLEDGEMENTS

The authors wish to acknowledge the Research Management Centre of University Teknologi Malaysia for partly supporting this research through UTM international Doctorial Fellowship

(IDF). We also acknowledge the effort of the anonymous reviewers for all their useful suggestions.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. John EJ. Introduction to hydraulics and hydrology with applications for stormwater management 2ed: Delmar Thompson Learning;1997.
2. Jurgen G, Gary WB. A muskingum-cunge channel flow routing method for drainage networks. Us Army Corps of Engineer;1991.
3. MSMA. Urban storm water management manual for malaysia. Department of Irrigation and Drainage. Percetakan Nasional Malaysia Beehad;2000.
4. John WL. Optimal operation of multireservoir systems: State of the art review. Journal of Water Resources Planning and Management. 2004;130(2):19.
5. Quentin BT. Opitmizing retention basin network. Journal of Water Resources Plannig and Management. 2008;134(4).
6. Us Army Corps of Engineer. Engineering and design flood-runoff analysis-streamflow and reservoir routing. USA;1994.
7. Beck RW. Detention pond sizing issue paper. Northwest Hydraulic Consultants, 1997. Accessed 24 November 2012. Available: http://www.co.snohomish.wa.us/documents/Departments/PDS/Planning_Commision/2010/Miscellaneous/DetentionPondSizing.pdf
8. City of Champaign Manual of Practice. Detention basin standards. Champaign, 2002. Accessed 24 November 2012. Available: http://ci.champaign.il.us/cms/wp-content/uploads/2008/08/c23-chapter-23-detention-basin-standards.pdf
9. NCDENR. NCDENR stormwater BMP manual-wet detention basin, 2007. Accessed 24 November 2012. Available : http://www.ncsu.edu/ehs/environ/DWQ_StormwaterBMPmanual_001%5B1%5D.pdf
10. Imve VN, Kofi AD, Istvan Z. Hydrological dimensioning and operation of reservoir: Practical design concept and principles. Kluwer Academic Publisher;2002.
11. Yiping G, Barry JA. Hydrologic analysis of urbanizing watershed with probablistic models. Conference on Statistical Methods for Hydrologic Systems. 1989.
12. Yiping G, Barry JA. Hydrologic analysis of urban catchments with event-based probabilistic models 1. Runoff volume. Water Resources Research. 1998;34(12):12.
13. Yiping G, Barry JA. An analytical probabilistic approach to sizing flood control detention facilities. Water Resources Research. 1999;35(8):12.
14. Yiping G, Barry JA. Analysis of detention ponds for storm water quality control. Water Resources Research. 1999;35(8):10.
15. Yiying X. Comparison of kinematic-wave and nonlinear reservoir routing of urban watershed runoff. Journal of Hydrologic Engineering. 2005;10(1):11.
16. Ramli MS, Raja IRMT, Ahmad MA, Mohamad NS, Mat Hussin MA. Improved coupled tank liquid levels system based on swarm adaptive tuning of hybrid proportional-integral neural network controller. American J. of Engineering and Applied Sciences. 2009;2(4):7.
17. Tomas H, Miroslav F. Dynamic optimization of hybrid coupled tanks system. Journal of Electrical Engineering. 2006;57(3):6.
18. Hassan KK. Nonlinear systems. 3 ed. upper saddle: Prentice Hall;2002.

19. Ronald RM. Nonlinear systems: Dynamic and control. Englewood Cliffs, New Jersey: Prentice Hall;1991.
20. Vidyasagar M. Nonlinear systems analysis. 3rd ed. Englewood Cliff, New Jersey: Prentice Hall;1993.
21. Thomas AS. Stormwater management for land development: Methods and calculations for quantity control. New Jersy: John Wiley & Sons, Inc.;2006.
22. Kaufman M and Gonze D. Theory of non-linear dynamical systems 2012 . Accessed 24 November 2012. Available: homepages.ulb.ac.be/~dgonze/TEACHING/nonlinear.pdf
23. Martin G, Micheal D. Linear algebra and differential equations using MATLAB. Brooks/Cole Publishing Company;1999.
24. Lefschetz S. Stability of nonlinear control systems. New York, London: Academy Press;1965.
25. Bermudez A, Martinez A. A state constrained optimal control problem related to the sterilization of canned foods. Automatica. 1994;30(2):319-29.
26. Anđelić MandFonseca CM. Sufficient conditions for positive definiteness of tridiagonal matrices revisited. Positivity. 2010;15(1):155-59.
27. Tisseur F. Tridiagonal-diagonal reduction of symmetric indefinite pairs. SIAM Journal on Matrix Analysis and Applications. 2004;26(1):215-32.
28. William WH, Roger NP. Norm-bounded tridiagonalizing similarity transformations for matrices: Carnegie Mellon University;1988:345.
29. Abdulrahman MKA, Mohd Ismail AA, Kek SL. Modelling of reservoir network for flood control using nonlinear reservoir routing model. International Journal of Engineering Science and Technology. 2012;4(7):9.
30. Liebe J, Van De Giesen N, Andreini M. Estimation of small reservoir storage capacities in a semi-arid environment: A case study in the upper east region of ghana. Physics and Chemistry of the Earth. 2005;30:7.

Peer-review history:
The peer review history for this paper can be accessed here:
http://www.sciencedomain.org/review-history.php?iid=160&id=5&aid=740