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A New Data Gathering Exponential Type Ratio Estimator for the Population Mean

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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ABSTRACT

In this article we proposed a new data gathering exponential ratio type estimator for the estimation of finite population mean under systematic sampling .The mean square error of the suggested estimator is computed up to the first degree of approximation and we find suggested estimator is efficient as compared with existing estimators. Furthermore this result is supported by numerical examples as well.

Keywords: Exponential ratio type estimator; systematic sampling; mean square error (MSE); efficiency.

1. INTRODUCTION

"In the literature of survey sampling, a simple technique of utilizing the known information of the population parameters of the auxiliary variables is through ratio, product, and regression method of estimations using different probability sampling designs such as simple random sampling, stratified random sampling, cluster sampling, systematic sampling, and double sampling. In the present paper we will use knowledge of the auxiliary variables under the framework of systematic sampling. Due to its simplicity, systematic sampling is the most

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commonly used probability design in survey of finite populations; see W. G. Madow and L. H. Madow" [1]. Apart from its simplicity, systematic sampling provides estimators which are more efficient than simple random sampling or stratified random Sampling for certain types of population.

Hajeck [2], Cochran [3] and Gautschi [4]. later on the problem of estimating the population mean using information on auxiliary variable has also been discussed by various authors including Quenouille [5], Hansen et al. [6], Swain [7], Banarasi et al. [8], Kadilar et, al. [9], Robson [10], Singh et, al. [11], Singh et, al. [12], Singh at, al. [13], Singh et al. [14], Kushwaha et, al.[15], Khan at, al. [16], Khan et, al [17], Singh [18], Shukla [19], Koyuncu et, al. [20], R. Singh et al. [21], R. Singh et al. [22], Bahl et, al. [23], Srivastava et, al. [24], Tailor et al. [25], and Ozel Kadilar, et, al. [26].

Consider a finite population $U = U_1, U_2, U_3 \dots\dots.. U_N$ of size N units. A sample of size n is taken at random from the first k units and every k^{th} subsequent unit then, N = nk where n and k are positive integers thus, there will be k samples each of size n and observe the study variate y and auxiliary variate x for each and every unit selected in the sample.

Let $\left(y_{ij,}x_{ij}\right) for\ i = 1.2 \dots..k$, $j = 1,2 \dots\dots..n$ indicate the value of j^{th} unit in the i^{th} sample. Then, the systematic sample means are defined as follows:

$\bar{y}_{st} = t_0 = 1/n \sum_{j=1}^{n} y_{ij,}$ and $\bar{x}_{st} = t_0 = 1/n \sum_{j=1}^{n} y_{ij,}$ unbiased estimators of the population means

$\bar{Y} = 1/n \sum_{j=1}^{n} y_{ij,}$ and $\bar{X} = 1/n \sum_{j=1}^{n} x_{ij,} of\ \ y\ \ on\ x$

To obtain estimators upto first order of approximation, using the following errors terms:

$$e_0 = \bar{y}_{sys} - \bar{Y}/\bar{Y}\ ,$$
$$e_1 = \bar{x}_{sys} - \bar{X}/\bar{X}\ ,$$
$$e_2 = \bar{z}_{sys} - \bar{Z}/\bar{Z}\ ,$$

$$Such\ that\ E(e_1) = 0\ for\ i = 0,1\ and\ 2$$

And

$$\rho_{yx} = \frac{s_{yx}}{s_y s_x}$$
$$\rho_{yz} = \frac{s_{yz}}{s_y s_z}$$
$$\rho_{xz} = \frac{s_{xz}}{s_x s_z}$$
$$k = \frac{\rho_{yx} C_y}{C_x}$$
$$k^* = \frac{\rho_{yz} C_y}{C_z}$$
$$\rho^*{}_y = \left\{1 + (n-1)\rho_y\right\}$$
$$\rho^*{}_x = \{1 + (n-1)\rho_x\}$$
$$\rho^*{}_z = \{1 + (n-1)\rho_z\}$$
$$\rho^{**} = \frac{\rho^*{}_y}{\rho^*{}_x}$$
$$\rho_2{}^{**} = \frac{\rho^*{}_y}{\rho^*{}_z}$$
$$\rho_1{}^{**} = \frac{\rho^*{}_y}{\rho^*{}_z}$$

Where

$\rho_y\ , \rho_x, \rho_z$ are intra class correlation among the pair of units for the variables y,x and z.

2. ESTIMATORS IN LITERATURE

In this part, we consider some estimators of the finite population mean in the sampling literature [27-29]. The variance and MSE's of all the estimators computed here are obtained upto first order of approximation.

The variance of the unbiased estimator for population mean is

$$var(t_0) = \lambda \bar{Y}^2 \varphi_0 \tag{1.1}$$

Swain [18] and Shukla [19] suggested the classical ratio and product estimators for finite population mean by are given by

$$t_1 = \bar{y}_{sy}\left(\frac{\bar{X}}{\bar{x}_{sy}}\right) \tag{1.2}$$

$$t_2 = \bar{y}_{sy} exp\left(\frac{\bar{z}_{sy}}{\bar{Z}}\right) \tag{1.3}$$

The mean square errors upto first orders of approximation are given as follows:

$$MSE(t_1) = \lambda\bar{Y}^2\left[\varphi_0 + \varphi_2\left(1 - 2k\sqrt{\rho^{**}}\right)\right] \tag{1.4}$$

$$\mathrm{MSE}(t_2) = \lambda\bar{Y}^2\left[\varphi_0 + \varphi_3\left(1 + 2k\sqrt{\rho_2{}^{**}}\right)\right] \tag{1.5}$$

Singh et al. [23] suggested the following ratio and product type exponential estimators as:

$$t_3 = \bar{y}_{sy} exp\left(\frac{\bar{X} - \bar{x}_{sy}}{\bar{X} + \bar{x}_{sy}}\right) \tag{1.6}$$

$$t_4 = \bar{y}_{sy} exp\left(\frac{\bar{x}_{sy} - \bar{X}}{\bar{x}_{sy} + \bar{X}}\right) \tag{1.7}$$

The mean square errors upto first orders of approximation are given as follows:

$$\mathrm{MSE}(t_3) = \lambda\bar{Y}^2\left[\varphi_0 + \frac{\varphi_2}{4}\left(1 - 4k\sqrt{\rho^{**}}\right)\right] \tag{1.8}$$

$$\mathrm{MSE}(t_4) = \lambda\bar{Y}^2\left[\varphi_0 + \frac{\varphi_2}{4}\left(1 + 4k\sqrt{\rho^{**}}\right)\right] \tag{1.9}$$

Tailor et al. [23] define the following ratio-cum product estimator as:

$$t_5 = \bar{y}_{sy}\left(\frac{\bar{X}}{\bar{x}_{sy}}\right)\left(\frac{\bar{z}_{sy}}{\bar{Z}}\right) \tag{2.0}$$

The MSE upto first order of approximation, is given by

$$\mathrm{MSE}(t_5) = \lambda\bar{Y}^2\left[\varphi_0 + \varphi_2\left(1 - 2k\sqrt{\rho^{**}}\right) + \varphi_3\left(1 - 2k^{**}\sqrt{\rho_1{}^{**}}\right) + 2\varphi_4\sqrt{\rho_y{}^*\rho_z{}^*}\right] \quad (2.1)$$

Where,

$\varphi_0 = \rho_y{}^* C_y{}^2$
$\varphi_1 = 2C_x{}^2\sqrt{\rho_y{}^*\rho_x{}^*}$
$\varphi_2 = \rho_x{}^* C_x{}^2$
$\varphi_3 = \rho_z{}^* C_z{}^2$
$\varphi_4 = k^* C^2{}_z$

2.1 Proposed Estimator

In this section, motivated Kadilar (2016) we proposed estimator for population mean under systematic sampling as given by :

$$t_{RK} = \bar{y}_{sy}\left(\frac{\bar{x}_{sy}}{\bar{X}}\right)^{\alpha} \exp\left(\frac{\bar{X} - \bar{x}_{sy}}{\bar{X} + \bar{x}_{sy}}\right) \tag{2.2}$$

To obtain estimators upto first order of approximation, using the following errors terms: $E(e_0{}^2) = \lambda\rho_y{}^* C_y{}^2$,

$E(e_1{}^2) = \rho_x{}^* C_x{}^2$
$E(e_0 e_1) = \lambda C^2{}_x\sqrt{\rho_y{}^*\rho_x{}^*}$

Where, $\lambda = \left(\frac{N-1}{nN}\right)$

Expressing (2.2) in terms of e's

$$t_{RK} = \bar{Y}(1+\varepsilon_0)(1+\varepsilon_1)^{\alpha} exp\left(\frac{\bar{X}-\bar{X}(1+\varepsilon_1)}{\bar{X}+\bar{X}(1+\varepsilon_1)}\right)$$

$$t_{RK} = \bar{Y}(1+\varepsilon_0)(1+\varepsilon_1)^{\alpha} exp\left[\frac{\varepsilon_1}{2}\left(1+\frac{\varepsilon_1}{2}\right)^{-1}\right] \tag{2.3}$$

$$t_{RK} = \bar{Y}(1+\varepsilon_0)(1+\varepsilon_1)^{\alpha} exp\left[\frac{\varepsilon_1}{2}\left(1+\frac{\varepsilon_1}{2}+\frac{{\varepsilon_1}^2}{4}+\cdots\right)\right]$$

$$t_{RK} = \bar{Y}(1+\varepsilon_0)\left(1+\alpha\varepsilon_1+\frac{\alpha(\alpha-1)}{2}{\varepsilon_1}^2+\cdots\right) exp\left[\frac{\varepsilon_1}{2}\left(1+\frac{\varepsilon_1}{2}+\frac{{\varepsilon_1}^2}{4}+\cdots\right)\right] \tag{2.4}$$

From (2.4)

$$t_{RK} - \bar{Y} \cong \bar{Y}\left[\alpha\varepsilon_1+\frac{\alpha(\alpha-1)}{2}{\varepsilon_1}^2+\frac{\varepsilon_1}{2}+\frac{\alpha{\varepsilon_1}^2}{2}+\frac{3{\varepsilon_1}^2}{8}+\varepsilon_0+\alpha\varepsilon_1\varepsilon_0+\frac{\varepsilon_0\varepsilon_1}{2}\right] \tag{2.5}$$

Squaring (2.5) on both sides and then taking expectation, the MSE of the estimator $\bar{y}_{RK}$

$$MSE(t_{RK}) = \lambda\bar{Y}^2\left[\varphi_0+\frac{K\varphi_1}{2}+\alpha K\varphi_1+(\alpha^2+\alpha)\frac{\varphi_2}{2}+\frac{\varphi_2}{8}\right] \tag{2.6}$$

Obtain the optimum α to minimize $MSE(\bar{y}_{RK})$. Differentiating $MSE(\bar{y}_{RK})$ w.r.t α and equating the derivative to zero. Optimum value of α is given by:

$$\alpha = -\frac{(K\varphi_1+\varphi_2)}{2\varphi_2}$$

Using the value of α_{opt} in (2.6), we get the minimum value of $\bar{y}_{RK}$

$$MSE_{min}(t_{RK}) = \lambda\bar{Y}^2\varphi_0\left[1-{\rho_{yx}}^2\right] \tag{2.7}$$

It follows from (2.7) that the proposed estimator $\bar{y}_{RK}$ at its optimum condition is equal efficient as that of the usual linear regression estimator.

2.2 Efficiency Comparisons

In this section, the MSE of traditional estimators t_0,t_1, t_2, t_3, $t_4$and $t_5$are compared with the MSE of proposed estimator $\bar{y}_{RK}$.

From (1.1) - (2.0) and (2.1)

$$[var(t_0) - MSE_{min}(\bar{y}_{RK})] > 0$$

$$\left[\lambda\bar{Y}^2\varphi_0{\rho_{yx}}^2\right] > 0 \tag{2.8}$$

$$[MSE(t_1) - MSE_{min}(\bar{y}_{RK})] > 0$$

$$\lambda\bar{Y}^2\left[\varphi_0\left(1-2k\sqrt{\rho^{**}}\right)\right] - \left[\lambda\bar{Y}^2\varphi_0{\rho_{yx}}^2\right] > 0 \tag{2.9}$$

$$[MSE(t_2) - MSE_{min}(\bar{y}_{RK})] > 0$$

$$\lambda\bar{Y}^2\left[\varphi_0\left(1+2k\sqrt{\rho^{**}}\right)\right] - \left[\lambda\bar{Y}^2\varphi_0{\rho_{yx}}^2\right] > 0 \tag{3.0}$$

$$[MSE(t_3) - MSE_{min}(\bar{y}_{RK})] > 0$$

$$\lambda\bar{Y}^2\left[\frac{\varphi_0}{4}\left(1-4k\sqrt{\rho^{**}}\right)\right] - \left[\lambda\bar{Y}^2\varphi_0{\rho_{yx}}^2\right] > 0 \tag{3.1}$$

$$[MSE(t_4) - MSE_{min}(\bar{y}_{RK})] > 0$$

$$\lambda\bar{Y}^2\left[\frac{\varphi_0}{4}\left(1+4k\sqrt{\rho^{**}}\right)\right] - \left[\lambda\bar{Y}^2\varphi_0{\rho_{yx}}^2\right] > 0 \tag{3.2}$$

$$[MSE(t_5) - MSE_{min}(\bar{y}_{RK})] > 0$$
$$\lambda\bar{Y}^2\left[\varphi_0\left(1-2k\sqrt{\rho^{**}}\right)+\varphi_3\left(1-2k^{**}\sqrt{\rho_1{}^{**}}\right)+2\varphi_4\sqrt{\rho_y{}^*\rho_z{}^*}\right]-\left[\lambda\bar{Y}^2\varphi_0\rho_{yx}{}^2\right] > 0 \tag{3.3}$$

Table 1. The percent relative efficiency of different estimators with respect to t_0

	Population	
Estimator	**MSE(t_α)**	$PRE(t_\alpha, t_0)$
t_0	1455.08	100.00
t_1	373.32	389.62
t_2	768.06	189.45
t_3	820.09	177.43
t_4	1044.42	139.32
t_5	187.08	777.79
t_{RK}	43.88	3316.04

2.3 Empirical Study

To examine the merits of the proposed estimator over the other existing estimators at optimum conditions, we have considered natural population data sets from the literature. The sources of population are given as follows.

(Source: Tailor et al. [23]). Consider Population

$N = 15$, $n = 3,$ $\bar{X} = 44.47, \bar{Y} = 80, \bar{Z} = 48.40, C_y = 0.56, C_x = 0.28, C_z = 0.43$
$S_y^2 = 2000, S_x^2 = 149.55, S_z^2 = 427.83, S_{yx} = 538.57, S_{yz} = -902.86, S_{xz} = -241.06,$
$\rho_{yx} = 0.9848, \rho_{yz} = -0.9760\rho_{xz} = -0.9530, \rho_y = 0.6652, \rho_x = 0.707, \rho_z = 0.5487.$

To find (PREs) of the estimator, we use the following formula:

$PRE(t_\alpha, t_0) = MSE(t_0)/MSE(t_\alpha) \times 100$

For $\alpha = 0,1,2,3,4,5$ and RK

3. CONCLUSION

A new data gathering exponential type estimator is proposed under systematic sampling and the properties of the suggested estimator are obtained up to first order of approximation. It has been seen that the suggested estimator performed better than the existing estimator both theoretically and empirically.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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