



Evolutionary Sequence of Spacetime and Intrinsic Spacetime and Associated Sequence of Geometries in Metric Force Fields III

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

A curved 'two-dimensional' absolute intrinsic metric spacetime $(\emptyset\hat{\rho}, \emptyset\hat{c}_s\emptyset\hat{t})$ on the vertical intrinsic spacetime hyperplane; its invariantly projected flat 'two-dimensional' absolute proper intrinsic metric spacetime $(\emptyset\rho'_{ab}, \emptyset c_{sab}\emptyset t'_{ab})$ and a flat 'two-dimensional' absolute proper metric spacetime $(\rho'_{ab}, c_{sab}t'_{ab})$, as the outward manifestation of the latter, evolve from a flat 'four-dimensional' absolute metric spacetime $(\hat{\mathbb{I}}\mathbb{E}^3, \hat{c}_s\hat{t})$ and its underlying flat 'two-dimensional' absolute intrinsic metric spacetime $(\emptyset\hat{\rho}, \emptyset\hat{c}_s\emptyset\hat{t})$, in all finite neighborhood of the source of a long-range metric force field. The flat four-dimensional relative proper metric spacetime $(\mathbb{I}\mathbb{E}'^3, c_{st'})$ and its underlying flat two-dimensional relative proper intrinsic metric spacetime $(\emptyset\rho', \emptyset c_s\emptyset t')$, remain unchanged within the field. The geometry is valid with respect to 3-observers located in the relative proper Euclidean 3-space $\mathbb{I}\mathbb{E}'^3$.

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A pair of absolute intrinsic metric tensor equations derived on the curved $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$ are solved algebraically to obtain the absolute intrinsic metric tensor and absolute intrinsic Ricci tensor on the curved $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$ in terms of an isolated absolute intrinsic geometrical parameter, referred to as absolute intrinsic 'static flow' speed, which the source of a long-range absolute intrinsic metric force field causes to be established on the extended curved $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$ from its location. This third part of this paper is the conclusion of the development of absolute intrinsic Riemann geometry on the curved 'two-dimensional' $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$ at the first stage of evolutions of spacetime and intrinsic spacetime in long-range metric force fields, started in the first and second parts. The first stage shows up as a numerical evolution. Extension to the second stage shall be done in the fourth and final part of this paper. Particularization to the gravitational field shall then follow in another article.

Keywords: *Long-range metric force fields; first stage of evolution of spacetime; numerical evolution of spacetime; absolute intrinsic Riemannian spacetime geometry; coexisting absolute intrinsic metric spacetimes; superposition procedure; resultant absolute intrinsic metric tensor; resultant absolute intrinsic Ricci tensor.*

1 INTRODUCTION

This third part of this paper is continuation of the derivation of absolute intrinsic Riemann geometry of curved absolute intrinsic Riemannian metric spacetime started in the first and second parts [1, 2]. As started with in [1], an initial flat 'four-dimensional' absolute metric spacetime $(\mathbb{I}\hat{\mathbb{E}}^3, \hat{c}_s\hat{t})$ is underlay by flat 'four-dimensional' absolute intrinsic metric spacetime $(\varnothing\hat{\mathbb{I}}\hat{\mathbb{E}}^3, \varnothing\hat{c}_s\varnothing\hat{t})$, where the flat 'three-dimensional' absolute metric space $\mathbb{I}\hat{\mathbb{E}}^3$ and its underlying flat 'three-dimensional' absolute intrinsic metric space $\varnothing\hat{\mathbb{I}}\hat{\mathbb{E}}^3$ lie as flat hyper-surfaces along the horizontal, while the straight line absolute metric time dimension $\hat{c}_s\hat{t}$ and straight line absolute intrinsic metric time dimension $\varnothing\hat{c}_s\varnothing\hat{t}$ lie along the vertical pseudo-orthogonal to the three-dimensional hyper-surfaces, with the assumed absence of absolute metric force field and absolute intrinsic force field.

The introduction of the source of a long-range absolute metric force field at a point in the absolute metric space is accompanied by the automatic location of the source of the counterpart absolute intrinsic metric force field at the corresponding point in the absolute intrinsic metric space. This follows from the perfect symmetry of state among the four symmetrical universes of the four-world picture, as explained in sub-section 3.1 of [3]. This action will cause the 'three-dimensional' absolute intrinsic metric space $\varnothing\hat{\mathbb{I}}\hat{\mathbb{E}}^3$ to be curved as a curved hyper-

surface toward the absolute intrinsic metric time dimension $\varnothing\hat{c}_s\varnothing\hat{t}$ along the vertical, thereby projecting flat absolute proper intrinsic metric space $\varnothing\mathbb{I}\hat{\mathbb{E}}_{ab}^{\prime 3}$ along the horizontal, which is made manifested outwardly in flat 'three-dimensional' absolute proper metric space $\mathbb{I}\hat{\mathbb{E}}_{ab}^{\prime 3}$, obtained by simply removing the symbol \varnothing —used to denote "intrinsic"—from $\varnothing\mathbb{I}\hat{\mathbb{E}}_{ab}^{\prime 3}$.

The initial flat 'three-dimensional' absolute metric space $\mathbb{I}\hat{\mathbb{E}}^3$ underlay by $\varnothing\hat{\mathbb{I}}\hat{\mathbb{E}}^3$ has transformed into flat $\mathbb{I}\hat{\mathbb{E}}_{ab}^{\prime 3}$ underly by flat $\varnothing\hat{\mathbb{I}}\hat{\mathbb{E}}_{ab}^{\prime 3}$, as a consequence of the introduction of the source of absolute metric force-field in the initial flat $\mathbb{I}\hat{\mathbb{E}}^3$ and the source of absolute intrinsic metric force-field in the initial flat $\varnothing\hat{\mathbb{I}}\hat{\mathbb{E}}^3$. This is accompanied by automatic appearance of flat three-dimensional relative proper metric space $\mathbb{I}\hat{\mathbb{E}}^{\prime 3}$ and its underlying flat three-dimensional relative proper intrinsic metric space $\varnothing\hat{\mathbb{I}}\hat{\mathbb{E}}^{\prime 3}$ in the geometry, such that observers are located in the flat relative proper (or physical) metric 3-space $\mathbb{I}\hat{\mathbb{E}}^{\prime 3}$. The projective straight line absolute proper intrinsic metric space $\varnothing\rho'_{ab}$ is embedded in the straight line relative proper intrinsic metric space $\varnothing\rho'$ that appears and $\varnothing\rho'_{ab}$, as outward manifestation of $\varnothing\rho'$, is embedded in $\mathbb{I}\hat{\mathbb{E}}^{\prime 3}$, as outward manifestation of $\varnothing\rho'$.

On the other hand, the absolute intrinsic metric time dimension $\varnothing\hat{c}_s\varnothing\hat{t}$ is not simultaneously curved with the absolute intrinsic metric space $\varnothing\hat{\mathbb{I}}\hat{\mathbb{E}}^3$, because it is unaffected by (or is invariant with) the presence of absolute intrinsic metric

force-field in the absolute intrinsic metric space. Thus an initial flat absolute metric spacetime $(\hat{\mathbb{E}}^3, \hat{c}_s \hat{t})$ and its underlay flat absolute intrinsic metric spacetime $(\emptyset \hat{\mathbb{E}}^3, \emptyset \hat{c}_s \emptyset \hat{t})$, evolve into the geometry of Figs. 6a and 6b of [1], reproduced as Figs. 1a and 1b here, as a consequence of the introduction of absolute metric force-field in an initial flat $\hat{\mathbb{E}}^3$ and absolute intrinsic metric force-field in the underlying initial flat $\emptyset \hat{\mathbb{E}}^3$. Figure 1b is the outward manifestation of the flat $(\emptyset \mathbb{E}_{ab}^3, \emptyset \hat{c}_s \emptyset \hat{t})$ in Fig. 1a. The two figures are one; they are separated for the sake of clarity only.

As developed in section 2 of the second part of this paper [2], the perfect isotropy of the absolute proper metric space \mathbb{E}_{ab}^3 , the absolute proper intrinsic metric space $\emptyset \mathbb{E}_{ab}^3$ and the curved absolute intrinsic metric space $\emptyset \hat{\mathbb{M}}^3$,

with respect to observers in the relative proper metric space \mathbb{E}^3 , causes them to be naturally contracted to 'one-dimensional absolute proper metric space ρ'_{ab} , one-dimensional absolute proper intrinsic metric space $\emptyset \rho'_{ab}$ and curved 'one-dimensional' absolute intrinsic metric space $\emptyset \hat{\rho}$, respectively, with respect to these observers. Consequently Figs. 6a and 6b of [1] become Fig. 6 of [2], presented as Fig. 2 of this article.

The absolute intrinsic Riemannian spacetime geometry being developed in this article and its preceding first and second parts is a pure novel effort of the author. No related work in physics or mathematics exists in the open literature, as far as can be found. This thereby limits the references in this paper to the previous papers of the author upon which it is based essentially.

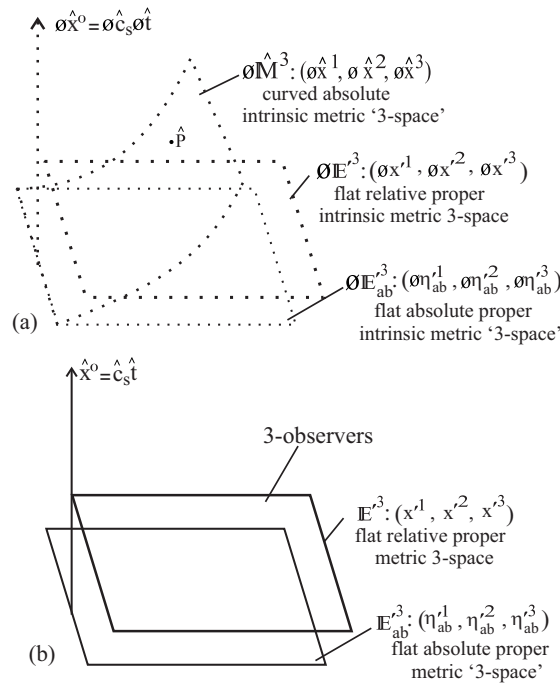


Fig. 1. (a) The '3-dimensional' absolute intrinsic metric space curving toward the absolute intrinsic metric time 'dimension' along the vertical, projects flat 3-dimensional absolute proper intrinsic metric space $\emptyset \mathbb{E}_{ab}^3$ along the horizontal and flat relative proper three-dimensional intrinsic metric space $\emptyset \mathbb{E}^3$ appears along the horizontal, (b) the flat $\emptyset \mathbb{E}_{ab}^3$ and $\emptyset \mathbb{E}^3$ in (a) are made manifested in flat 3-dimensional absolute proper metric space \mathbb{E}_{ab}^3 , where observers are located in \mathbb{E}^3 ; (Figs. 6a&b of [1])

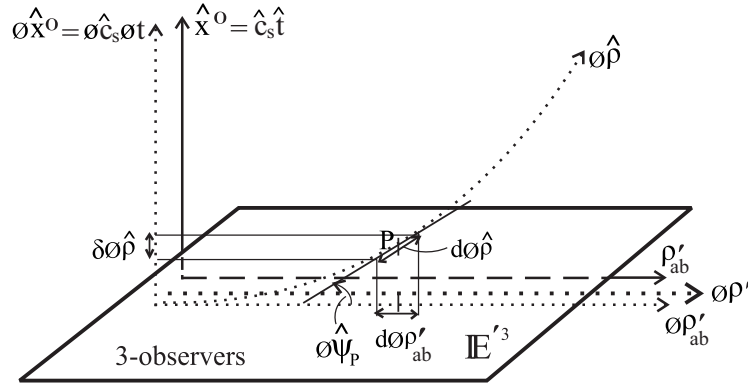


Fig. 2. A curved ‘one-dimensional’ absolute intrinsic metric space $\hat{\rho}$, curving toward the absolute time/absolute intrinsic time ‘dimensions’ along the vertical, projects a straight line one-dimensional isotropic absolute proper intrinsic metric space $\hat{\rho}'_{ab}$ along the horizontal, which is made manifested in straight line absolute proper metric space ρ'_{ab} , and straight line relative proper intrinsic metric space $\hat{\rho}'$ appears, which is made manifested in flat three-dimensional relative metric space \mathbb{E}'^3 (as a hyper-surface) along the horizontal, with respect to 3-observers in \mathbb{E}'^3 in our (or positive) universe; (Fig. 6 of [2])

2 INCLUSION OF CURVED ABSOLUTE INTRINSIC METRIC TIME ‘DIMENSION’

One crucial feature of Fig. 2 is that the absolute intrinsic metric time ‘dimension’, $\hat{x}^0 \equiv \hat{c}_s \hat{t}$, is not curved along with the absolute intrinsic metric space $\hat{\rho}$, with respect to 3-observers in \mathbb{E}'^3 . This, as shall be explained with further development in this article, is due to the fact that the presence of a long-range absolute intrinsic metric force field on an initially flat ‘two-dimensional’ absolute intrinsic metric spacetime $(\hat{\rho}, \hat{c}_s \hat{t})$ (that is, $\hat{\rho}$ is a straight line along the horizontal and $\hat{c}_s \hat{t}$ is a straight line along the vertical), underlying an initially flat absolute metric spacetime $(\mathbb{E}^3, \hat{c}_s \hat{t})$ (as shall be illustrated later in this article), which causes curved absolute intrinsic metric space $\hat{\rho}$ to evolve within the absolute metric force field (as in Fig. 2), does not give rise to simultaneous curvature of the absolute intrinsic metric time ‘dimension’ $\hat{c}_s \hat{t}$ from its vertical position, with respect to 3-observers in the relative proper Euclidean metric 3-space \mathbb{E}'^3 . This is the geometrical interpretation of the fact that absolute time and

absolute intrinsic time are invariant (or remain unchanged) and, hence, do not transform into absolute proper metric time and absolute proper intrinsic metric time respectively, with respect to 3-observers in \mathbb{E}'^3 , in the context of absolute physics/absolute intrinsic physics.

The feature of the geometry of Fig. 2 discussed in the preceding paragraph makes the absolute intrinsic line element to take on the Gaussian form of Eq. (89) of part two of this paper [2], on the curved absolute intrinsic metric space $\hat{\rho}$, with respect to 3-observers in the underlying relative proper metric Euclidean 3-space \mathbb{E}'^3 , which shall be re-produced here as follows

$$(d\hat{s})^2 = (d\hat{x}^0)^2 - \hat{g}_{11}(d\hat{\rho})^2, \quad (1)$$

where

$$\hat{g}_{11} = \sec^2 \hat{\psi}_P = (1 - \hat{k}_P^2)^{-1}; \quad (2)$$

$\hat{\psi}_P$ is the absolute intrinsic angle of inclination of the curved absolute intrinsic metric space $\hat{\rho}$ to its projective absolute proper intrinsic metric space $\hat{\rho}'_{ab}$ along the horizontal, at point P along $\hat{\rho}$; \hat{k}_P is the absolute intrinsic curvature parameter at point P along the curved $\hat{\rho}$ and $\hat{k}_P = \sin \hat{\psi}_P$, as derived in in sub-section 1.1 of part two of this paper [2].

As developed in sub-section 1.3 of [3], the relative proper metric time dimension $c_s t'$ and its underlying relative proper intrinsic metric time dimension $\varnothing c_s \varnothing t'$ of our (or positive) universe are actually the relative proper metric Euclidean 3-space $\mathbb{E}^{0/3}$ and the one-dimensional relative proper intrinsic metric space $\varnothing \rho^{0'}$ of the positive time-universe. And the relative proper metric Euclidean 3-space \mathbb{E}'^3 and the relative proper intrinsic metric space $\varnothing \rho'$ of our universe are the relative proper metric time dimension $c_s t^{0'}$ and its underlying relative proper intrinsic metric time dimension $\varnothing c_s \varnothing t^{0'}$ of the positive time-universe. There exist perfect symmetry of state and perfect symmetry of natural laws between the positive (or our) universe and the positive time-universe and, indeed among the four symmetrical universes isolated in [3–6], as demonstrated in section 2 of [5], section 2 of [3] and section 2 of [6].

Perfect symmetry of state among the four universes implies that, corresponding to the half-geometry of Fig. 2 that evolves at the first stage of evolutions of spacetime/intrinsic spacetime within a long-range metric force field in our universe, there is an identical half-geometry that evolves simultaneously at the first stage of evolution of spacetime/intrinsic spacetime within the identical symmetry-partner long-range metric force field in each of the other three universes. The identical half-geometry in the positive time-universe, depicted in Fig. 3, which is valid with respect to 3-observers⁰ in the relative proper metric Euclidean 3-space $\mathbb{E}^{0/3}$ of the positive time-universe, coexists with the half-geometry of Fig.2 in our universe, which is valid with respect to 3-observers in the relative proper metric Euclidean 3-space \mathbb{E}'^3 of our (or positive) universe.

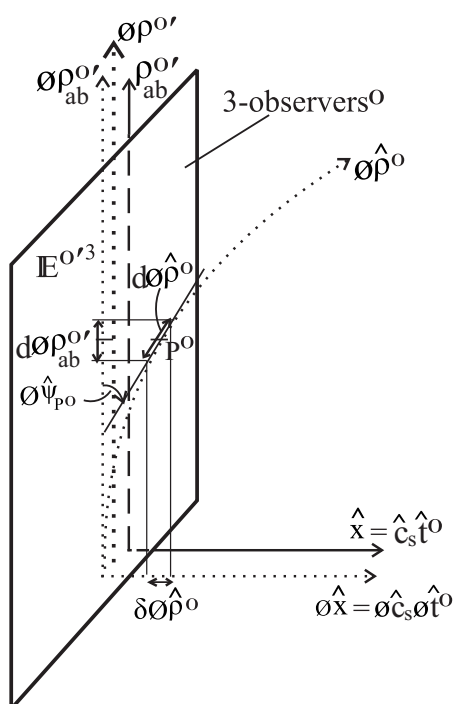


Fig. 3. A curved ‘one-dimensional’ absolute intrinsic metric space $\varnothing \hat{\rho}^0$, curving toward the absolute metric time/absolute intrinsic metric time ‘dimensions’ along the horizontal, projects a straight line one-dimensional isotropic absolute proper intrinsic metric space $\varnothing \rho^{0'}/_{ab}$ underneath the relative proper metric Euclidean 3-space $\mathbb{E}^{0/3}$ along the vertical, with respect to 3-observers in $\mathbb{E}^{0/3}$ in the positive time-universe

The absolute intrinsic line element is given at point P^0 on the curved absolute intrinsic metric space $\varnothing\hat{\rho}^0$, with respect to 3-observers in the relative proper metric Euclidean 3-space \mathbb{E}^{0r3} in Fig. 3 (like Eq. (1) at the symmetry-partner point P on the curved absolute intrinsic metric space $\varnothing\hat{\rho}$, with respect to 3-observers in \mathbb{E}^{r3} in Fig. 2) as

$$(d\varnothing\hat{s}^0)^2 = (d\varnothing\hat{x}^1)^2 - \varnothing\hat{g}_{00}(d\varnothing\hat{\rho}^0)^2, \quad (3)$$

where

$$\varnothing\hat{g}_{00} = \sec^2 \varnothing\hat{\psi}_{P^0} = (1 - \varnothing\hat{k}_{P^0}^2)^{-1}; \quad (4)$$

$\varnothing\hat{\psi}_{P^0}$ is the absolute intrinsic angle of inclination of the curved absolute intrinsic metric space $\varnothing\hat{\rho}^0$ to its projection $\varnothing\rho_{ab}^{0r}$ along the vertical, at point P^0 along $\varnothing\hat{\rho}^0$; $\varnothing\hat{k}_{P^0}$ is the absolute intrinsic

curvature parameter of the curved $\varnothing\hat{\rho}^0$ at point P^0 along $\varnothing\hat{\rho}^0$ and $\varnothing\hat{k}_{P^0} = \sin \varnothing\hat{\psi}_{P^0}$.

Fig. 3 in the positive time-universe is half-geometry, just as Fig.2 in our universe is half-geometry. These half-geometries co-exist and must be united into the singular full geometry depicted in Fig. 4. The absolute metric time 'dimension' $\hat{c}_s\hat{t}$ and absolute intrinsic metric time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ along the vertical in Fig.2 do not appear in Fig. 4, having been supplanted by $\varnothing\rho_{ab}^{0r}$ and $\varnothing\rho_{ab}^{0r}$ respectively, and the absolute metric time 'dimension' $\hat{c}_s\hat{t}^0$ and absolute intrinsic metric time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}^0$ along the horizontal in Fig. 3 do not appear in Fig. 4, having been supplanted by ρ'_{ab} and $\varnothing\rho'_{ab}$ respectively.

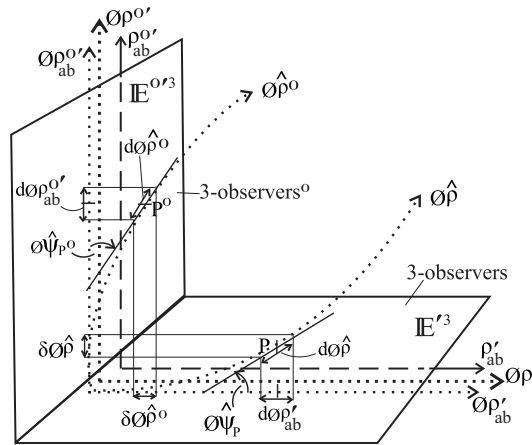


Fig. 4. Curved 'two-dimensional' absolute intrinsic metric space ($\varnothing\hat{\rho}, \varnothing\hat{\rho}^0$) and its projective flat 'two-dimensional' absolute proper intrinsic metric space ($\varnothing\rho'_{ab}, \varnothing\rho_{ab}^{0r}$) underlying flat six-dimensional relative proper physical space ($\mathbb{E}^{r3}, \mathbb{E}^{0r3}$) with respect to 3-observers in \mathbb{E}^{r3} in our universe and 3-observers in \mathbb{E}^{0r3} in the positive time-universe obtained by uniting Fig. 2 and Fig. 3

The projection of the elementary absolute intrinsic metric space coordinate interval $d\varnothing\hat{\rho}$ about point P along $\varnothing\hat{\rho}$ into the horizontal and the projection of the corresponding elementary intrinsic metric space coordinate interval $d\varnothing\hat{\rho}^0$ about point P^0 along the curved $\varnothing\hat{\rho}^0$ into the vertical in Fig. 4 are given respectively as

$$d\varnothing\rho'_{ab} = d\varnothing\hat{\rho} \cos \varnothing\hat{\psi}_P; \quad (\text{w.r.t. 3 - observers in } \mathbb{E}^{r3}) \quad (5a)$$

and

$$d\varnothing\rho_{ab}^{0r} = d\varnothing\hat{\rho}^0 \cos \varnothing\hat{\psi}_{P^0}; \quad (\text{w.r.t. 3 - observers in } \mathbb{E}^{0r3}) \quad (5b)$$

The ‘non-metric’ component, $\delta\varnothing\hat{\rho} = d\varnothing\hat{\rho} \sin \varnothing\hat{\psi}_P$, projected into $\varnothing\hat{c}_s\varnothing\hat{t}$ along the vertical by interval $d\varnothing\hat{\rho}$ about point P of curved $\varnothing\hat{\rho}$ in Fig. 2, is now projected into the absolute proper intrinsic metric space $\varnothing\rho'_{ab}$ along the vertical in Fig. 4, and the ‘non-metric’ component, $\delta\varnothing\hat{\rho}^0 = d\varnothing\hat{\rho}^0 \sin \varnothing\hat{\psi}_{P^0}$, projected into $\varnothing\hat{c}_s\varnothing\hat{t}^0$ along the horizontal by interval $d\varnothing\hat{\rho}^0$ about point P^0 along the curved $\varnothing\hat{\rho}^0$ in Fig. 3, is now projected into the absolute proper intrinsic metric space $\varnothing\rho'_{ab}$ along the horizontal in Fig. 4.

Although the ‘non-metric’ components $\delta\varnothing\hat{\rho}$ and $\delta\varnothing\hat{\rho}^0$ actually exist as shown in Fig. 4, they cannot appear in the intrinsic metric coordinate interval projection expressions (5a) and (5b). They cannot be considered at equal pedestal with the projective absolute proper intrinsic metric coordinate intervals with the prime label.

However let us temporarily take into account the projective ‘non-metric’ components in the intrinsic coordinate projections that can be derived from Fig. 4 to have the following

$$(d\varnothing\rho'_{ab}) = (d\varnothing\hat{\rho}) \cos \varnothing\hat{\psi}_P; \text{ and } \delta\varnothing\hat{\rho}^0 = d\varnothing\hat{\rho}^0 \sin \varnothing\hat{\psi}_{P^0};$$

(w.r.t. 3 – observers in \mathbb{E}^{r3}); (6a)

$$d\varnothing\rho^0_{ab} = d\varnothing\hat{\rho}^0 \cos^2 \varnothing\hat{\psi}_{P^0}; \text{ and } \delta\varnothing\hat{\rho} = d\varnothing\hat{\rho} \sin \varnothing\hat{\psi}_P;$$

(w.r.t. 3 – observers in \mathbb{E}^{0r3}). (6b)

Now there is equality of the square of intrinsic coordinate interval $d\varnothing\hat{\rho}^2$ along the curved absolute intrinsic metric space $\varnothing\hat{\rho}$ and the sum of squares of the intrinsic metric coordinate intervals, $d\varnothing\rho'_{ab}$ and $\delta\varnothing\hat{\rho}^0$, along the straight line absolute proper intrinsic metric space $\varnothing\rho'_{ab}$, which are projected along the horizontal by $d\varnothing\hat{\rho}$ and $d\varnothing\hat{\rho}^0$ respectively in Fig. 4, expressed as follows

$$(d\varnothing\hat{\rho})^2 = (d\varnothing\rho'_{ab})^2 + (\delta\varnothing\hat{\rho}^0)^2,$$

or

$$(d\varnothing\rho'_{ab})^2 = (d\varnothing\hat{\rho})^2 - (\delta\varnothing\hat{\rho}^0)^2.$$

This can be seen as invariance of partial intrinsic ‘line element’ between the curved $\varnothing\hat{\rho}$ and its projective straight line $\varnothing\rho'_{ab}$ along the horizontal with respect to 3-observers in \mathbb{E}^{r3} in Fig. 4.

Using system (6a) in the last displayed equation gives,

$$(d\varnothing\rho'_{ab})^2 = (d\varnothing\rho'_{ab})^2 \sec^2 \varnothing\hat{\psi}_P - (d\varnothing\hat{\rho}^0)^2 \sin^2 \varnothing\hat{\psi}_{P^0}.$$

This simplifies further as follows by virtue of Eq. (5b),

$$(d\varnothing\rho'_{ab})^2 = (d\varnothing\rho'_{ab})^2 \sec^2 \varnothing\hat{\psi}_P - (d\varnothing\rho^0_{ab})^2 \tan^2 \varnothing\hat{\psi}_{P^0};$$

(w.r.t. 3 – observers in \mathbb{E}^{r3}). (7a)

There is likewise invariance of partial intrinsic line element between the the curved absolute intrinsic metric space $\varnothing\hat{\rho}^0$ and its projective straight line absolute proper intrinsic metric space $\varnothing\rho^0_{ab}$ along the vertical in Fig. 4, expressed as follows

$$(d\varnothing\hat{\rho}^0)^2 = (d\varnothing\rho^0_{ab})^2 + (\delta\varnothing\hat{\rho})^2,$$

or

$$(d\varnothing\rho^0_{ab})^2 = (d\varnothing\hat{\rho}^0)^2 - (\delta\varnothing\hat{\rho})^2,$$

which upon using system (6b) gives

$$(d\varnothing\rho^0_{ab})^2 = (d\varnothing\rho^0_{ab})^2 \sec^2 \varnothing\hat{\psi}_{P^0} - (d\varnothing\hat{\rho})^2 \sin^2 \varnothing\hat{\psi}_P.$$

This simplifies further as follows by virtue of Eq. (5a)

$$(d\varnothing\rho_{ab}^{0'})^2 = (d\varnothing\rho_{ab}^{0'})^2 \sec^2 \varnothing\hat{\psi}_{P^0} - (d\varnothing\rho_{ab}')^2 \tan^2 \varnothing\hat{\psi}_P; \quad (7b)$$

(w.r.t. 3 – observers in $\mathbb{E}^{0'3}$).

Point P along the curved $\varnothing\hat{\rho}$ and point P^0 along the curved $\varnothing\hat{\rho}^0$ are symmetry-partner points. Consequently the absolute intrinsic angles, $\varnothing\hat{\psi}_P$ and $\varnothing\hat{\psi}_{P^0}$, are equal. We can therefore let, $\varnothing\hat{\psi}_P = \varnothing\hat{\psi}_{P^0} \equiv \varnothing\hat{\psi}$. By using this fact and adding Eqs. (7a) and (7b) we have

$$\begin{aligned} (d\varnothing\rho_{ab}^{0'})^2 + (d\varnothing\rho_{ab}')^2 &= (d\varnothing\rho_{ab}^{0'})^2 (\sec^2 \varnothing\hat{\psi} - \tan^2 \varnothing\hat{\psi}) \\ &\quad + (d\varnothing\rho_{ab}')^2 (\sec^2 \varnothing\hat{\psi} - \tan^2 \varnothing\hat{\psi}) \end{aligned} \quad (8)$$

Equation (8) expresses absolute intrinsic local Euclidean invariance (A \varnothing LEI) in terms of absolute proper intrinsic metric coordinate intervals, partially with respect to 3-observers in \mathbb{E}'^3 and partially with respect to 3-observers⁰ in $\mathbb{E}^{0'3}$, by virtue of the expression, $\sec^2 \varnothing\hat{\psi} - \tan^2 \varnothing\hat{\psi} = 1$.

The full invariance of intrinsic line element (8) between the curved ‘two-dimensional’ absolute intrinsic metric space ($\varnothing\hat{\rho}, \varnothing\hat{\rho}^0$) and the flat two-dimensional absolute proper intrinsic metric space ($\varnothing\rho_{ab}^{0'}, \varnothing\rho_{ab}'$), with respect to 3-observers in \mathbb{E}'^3 and 3-observers⁰ in $\mathbb{E}^{0'3}$, has been written partially as invariance of intrinsic line element (7a) between the curved $\varnothing\hat{\rho}$ and its projective straight line $\varnothing\rho_{ab}'$ with respect to 3-observers in \mathbb{E}'^3 and partially as invariance of intrinsic line element (7b) between the curved $\varnothing\hat{\rho}^0$ and its projective straight line $\varnothing\rho_{ab}^{0'}$ with respect to 3-observers⁰ in $\mathbb{E}^{0'3}$ in Fig. 4 earlier.

Now the invariance of intrinsic line element between the curved ‘two-dimensional’ absolute intrinsic metric space ($\varnothing\hat{\rho}, \varnothing\hat{\rho}^0$) and its projective flat ‘two-dimensional’ absolute proper intrinsic metric space ($\varnothing\rho_{ab}', \varnothing\rho_{ab}^{0'}$) in Fig. 4 can be expressed as

$$(d\varnothing\hat{s}^*)^2 = (d\varnothing s'^*)^2,$$

or

$$(d\varnothing\hat{\rho}^0)^2 + (d\varnothing\hat{\rho})^2 = (d\varnothing\rho_{ab}^{0'})^2 + (d\varnothing\rho_{ab}')^2. \quad (9)$$

It then follows that the absolute proper intrinsic metric space intervals $d\varnothing\rho_{ab}^{0'}$ and $d\varnothing\rho_{ab}'$ can be replaced with the absolute intrinsic metric space intervals $d\varnothing\hat{\rho}^0$ and $d\varnothing\hat{\rho}$ respectively in Eq. (8) to have

$$\begin{aligned} (d\varnothing\hat{\rho}^0)^2 + (d\varnothing\hat{\rho})^2 &= (d\varnothing\hat{\rho}^0)^2 (\sec^2 \varnothing\hat{\psi} - \tan^2 \varnothing\hat{\psi}) \\ &\quad + (d\varnothing\hat{\rho})^2 (\sec^2 \varnothing\hat{\psi} - \tan^2 \varnothing\hat{\psi}) \end{aligned} \quad (10)$$

Equation (10) expresses intrinsic local Euclidean invariance on the curved ‘two-dimensional’ absolute intrinsic metric space ($\varnothing\hat{\rho}, \varnothing\hat{\rho}^0$) in terms of absolute intrinsic coordinate intervals, partially with respect to 3-observers in \mathbb{E}'^3 and partially with respect to 3-observers in $\mathbb{E}^{0'3}$, by virtue of relation, $\sec^2 \varnothing\hat{\psi} - \tan^2 \varnothing\hat{\psi} = 1$. Let us replace $(d\varnothing\hat{\rho}^0)^2 + (d\varnothing\hat{\rho})^2$ by the square of the starred absolute intrinsic Euclidean line element $(d\varnothing\hat{s}^*)^2$ at the left-hand side of Eq. (10) to have

$$(d\varnothing\hat{s}^*)^2 = (d\varnothing\hat{\rho}^0)^2 (\sec^2 \varnothing\hat{\psi} - \tan^2 \varnothing\hat{\psi}) + (d\varnothing\hat{\rho})^2 (\sec^2 \varnothing\hat{\psi} - \tan^2 \varnothing\hat{\psi}), \quad (11)$$

or

$$(d\varnothing\hat{s}^*)^2 = (d\varnothing\hat{\rho}^0)^2 + (d\varnothing\hat{\rho})^2. \quad (12)$$

The need for the star label on the absolute intrinsic line element $d\varnothing\hat{s}^*$ shall be seen later in this article. The absolute intrinsic Euclidean line element (11) or (12) obtains at every point along the curved $\varnothing\hat{\rho}$ and at the symmetry-partner point along the curved $\varnothing\hat{\rho}^0$, partially with respect to 3-observers in \mathbb{E}'^3

and partially with respect to 3-observers in $\mathbb{E}^{0'}$ in Fig. 4, in so far as both the metric and 'non-metric' intrinsic coordinate interval projections are taken into account in deriving intrinsic coordinate interval projection relations from Fig. 4, as done in systems (6a) and (6b) and Eqs. (7a) and (7b).

Now let us as done on 'two-dimensional' and 'three-dimensional' absolute intrinsic metric spaces $\mathcal{O}\hat{\mathbb{M}}^2$ and $\mathcal{O}\hat{\mathbb{M}}^3$ in sub-section 1.1 of [2], separate the absolute intrinsic Euclidean line element $(d\mathcal{O}\hat{s}^*)^2$ of Eq. (11) into the metric and non-metric components, $(d\mathcal{O}\hat{s}_m^*)^2$ and $(d\mathcal{O}\hat{s}_{nm}^*)^2$, as follows

$$\begin{aligned} (d\mathcal{O}\hat{s}^*)^2 &= (d\mathcal{O}\hat{s}_m^*)^2 + (d\mathcal{O}\hat{s}_{nm}^*)^2 \\ &= \sum_{i,j=0}^1 \mathcal{O}\hat{g}_{ij}^* d\mathcal{O}\hat{x}^i d\mathcal{O}\hat{x}^j - \sum_{i,j=0}^1 \mathcal{O}\hat{R}_{ij}^* d\mathcal{O}\hat{x}^i d\mathcal{O}\hat{x}^j \end{aligned} \quad (13)$$

$$\begin{aligned} (d\mathcal{O}\hat{s}^*)^2 &= \left(\sec^2 \mathcal{O}\hat{\psi} (d\mathcal{O}\hat{\rho}^0)^2 + \sec^2 \mathcal{O}\hat{\psi} (d\mathcal{O}\hat{\rho})^2 \right) \\ &\quad - \left(\tan^2 \mathcal{O}\hat{\psi} (d\mathcal{O}\hat{\rho}^0)^2 + \tan^2 \mathcal{O}\hat{\psi} (d\mathcal{O}\hat{\rho})^2 \right). \end{aligned} \quad (14)$$

The absolute intrinsic metric line element on the curved 'two-dimensional' absolute intrinsic metric space $(\mathcal{O}\hat{\rho}, \mathcal{O}\hat{\rho}^0)$, which is valid partially with respect to 3-observers in \mathbb{E}^{r3} and partially with respect to 3-observers in \mathbb{E}^{0r3} in Fig. 4, which follows from Eqs. (13) and (14) is the following

$$\begin{aligned} (d\mathcal{O}\hat{s}_m^*)^2 &= \sum_{i,j=0}^1 \mathcal{O}\hat{g}_{ij}^* d\mathcal{O}\hat{x}^i d\mathcal{O}\hat{x}^j \\ &= \mathcal{O}\hat{g}_{00}^* (d\mathcal{O}\hat{\rho}^0)^2 + \mathcal{O}\hat{g}_{11}^* (d\mathcal{O}\hat{\rho})^2; \end{aligned} \quad (15)$$

$$= \sec^2 \mathcal{O}\hat{\psi} (d\mathcal{O}\hat{\rho}^0)^2 + \sec^2 \mathcal{O}\hat{\psi} (d\mathcal{O}\hat{\rho})^2; \quad (16)$$

$$= \frac{(d\mathcal{O}\hat{\rho}^0)^2}{1 - \mathcal{O}\hat{k}^2} + \frac{(d\mathcal{O}\hat{\rho})^2}{1 - \mathcal{O}\hat{k}^2}. \quad (17)$$

The implied absolute intrinsic metric tensor is

$$\mathcal{O}\hat{g}_{ij}^* = \begin{pmatrix} \sec^2 \mathcal{O}\hat{\psi} & 0 \\ 0 & \sec^2 \mathcal{O}\hat{\psi} \end{pmatrix} = \begin{pmatrix} \frac{1}{1 - \mathcal{O}\hat{k}^2} & 0 \\ 0 & \frac{1}{1 - \mathcal{O}\hat{k}^2} \end{pmatrix}. \quad (18)$$

The derived circular absolute intrinsic metric line element (16) or (17) is the absolute intrinsic line element on the curved 'two-dimensional' absolute intrinsic metric space $(\mathcal{O}\hat{\rho}, \mathcal{O}\hat{\rho}^0)$ in Fig. 4. It is effectively the union of the partial absolute intrinsic line element (1) derived with respect to 3-observers in the relative proper metric Euclidean 3-space \mathbb{E}^{r3} from Fig. 2 and partial absolute intrinsic line element (3) derived with respect to 3-observers⁰ in the relative proper metric Euclidean 3-space \mathbb{E}^{0r3} from Fig. 3, just as Fig. 3 from which (16) or (17) has been derived is the union of Figs. 2 and 3.

It is to be noted, as explicitly stated by Eqs. (1) and (3) that, the term, $\mathcal{O}\hat{g}_{00}^* (d\mathcal{O}\hat{\rho}^0)^2 = (d\mathcal{O}\hat{\rho}^0)^2 / (1 - \mathcal{O}\hat{k}^2)$, of the absolute intrinsic line element (17) has been derived by and is hence valid with respect to 3-observers⁰ in the relative proper metric Euclidean 3-space \mathbb{E}^{0r3} of the positive time-universe, while the term, $\mathcal{O}\hat{g}_{11}^* (d\mathcal{O}\hat{\rho})^2 = (d\mathcal{O}\hat{\rho})^2 / (1 - \mathcal{O}\hat{k}^2)$, has been derived by and is hence valid with respect to 3-observers in the relative proper physical Euclidean 3-space \mathbb{E}^{r3} of our (or positive) universe in Fig. 4. Thus the components, $\mathcal{O}\hat{g}_{00}^*$ and $\mathcal{O}\hat{g}_{11}^*$, of the derived circular absolute intrinsic metric tensor $\mathcal{O}\hat{g}_{ij}^*$ of Eq. (18) are valid with respect to 3-observers in \mathbb{E}^{0r3} and \mathbb{E}^{r3} respectively.

In effect, the curved 'two-dimensional' absolute intrinsic metric space $(\mathcal{O}\hat{\rho}, \mathcal{O}\hat{\rho}^0)$ is an absolute intrinsic Riemannian metric manifold without curved absolute intrinsic metric time 'dimension' (i.e. it is of the

class $\varnothing\hat{M}^p$; $p = 2$), which is underlay by its projective flat ‘two-dimensional’ absolute proper intrinsic metric space $(\varnothing\rho'_{ab}, \varnothing\rho^0_{ab})$ and the flat six-dimensional relative proper metric space $(\mathbb{E}'^3, \mathbb{E}^{0/3})$ in which the observers are located. This is so, since curved absolute intrinsic metric time ‘dimension’ does not exist with respect to either the 3-observers in \mathbb{E}'^3 or 3-observers⁰ in $\mathbb{E}^{0/3}$ in Fig. 4, who jointly construct the absolute intrinsic line element (16) or (17).

One important consequence of the perfect symmetry of state between our (or positive) universe and the positive time-universe is that, the absolute intrinsic metric line element (1), written at point P on the curved ‘one-dimensional’ absolute intrinsic metric space $\varnothing\hat{\rho}$, by 3-observers in the relative proper Euclidean 3-space \mathbb{E}'^3 of our universe, in the half-geometry of Fig.2, is perfectly identical to the absolute intrinsic metric line element (3), written at the symmetry-partner point P⁰ on the curved $\varnothing\hat{\rho}^0$, by 3-observers⁰ in the relative proper Euclidean 3-space $\mathbb{E}^{0/3}$ of the positive time-universe, in the half-geometry of Fig. 3. In other words, the component $\varnothing\hat{g}^*_{11}$ of the absolute intrinsic metric tensor in Eq. (1) is identical to the component $\varnothing\hat{g}^*_{00}$ in Eq. (3).

Having derived Fig. 4 and the absolute intrinsic line element (16) or (17) on the curved ‘two-dimensional’ absolute intrinsic metric space $(\varnothing\hat{\rho}, \varnothing\hat{\rho}^0)$, partially with respect to 3-observers in \mathbb{E}'^3 and partially with respect to 3-observers⁰ in $\mathbb{E}^{0/3}$ in that figure, let us now modify both the figure and the absolute intrinsic line element to the forms in which they are valid for absolute intrinsic Riemann geometry in our universe solely. This shall be done in two steps. At the first step, we recognize that the dimensions, $x^{0/1}$, $x^{0/2}$ and $x^{0/3}$, of the relative proper metric Euclidean 3-space $\mathbb{E}^{0/3}$, the absolute proper intrinsic metric space $\varnothing\rho^0_{ab}$ and absolute intrinsic metric space $\varnothing\hat{\rho}^0$ of the positive time-universe in Fig. 4, are elusive to 3-observers in the relative proper metric Euclidean 3-space \mathbb{E}'^3 of our universe, hence they cannot appear in physics in our universe.

As developed in sub-section 1.3 of [3], the relative proper metric Euclidean 3-space $\mathbb{E}^{0/3}$ of the positive time-universe induces the relative proper metric time dimension $c_s t'$ of our universe, with respect to all 3-observers in the relative proper Euclidean 3-space \mathbb{E}'^3 of our universe. Thus in converting Fig. 4 to the form it will be useful in our universe, we must first of all let $\mathbb{E}^{0/3} \rightarrow c_s t'$, $\varnothing\rho^0 \rightarrow \varnothing c_s \varnothing t'$ and $\varnothing\hat{\rho}^0 \rightarrow \varnothing\hat{c}_s \varnothing\hat{t}$ (as done in a similar situation with system (15) of [6]), in the upper half of Fig. 4 to have Fig. 5.

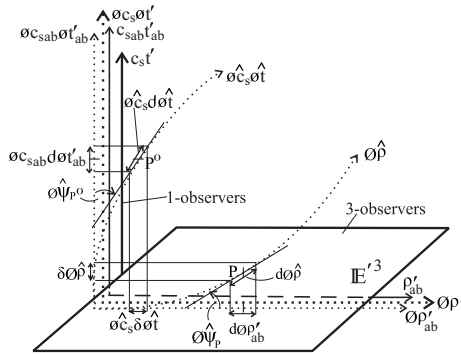


Fig. 5. Curved ‘two-dimensional’ absolute intrinsic metric spacetime $(\varnothing\hat{\rho}, \varnothing\hat{c}_s \varnothing\hat{t})$, its projective flat two-dimensional absolute proper intrinsic metric spacetime $(\varnothing\rho'_{ab}, \varnothing c_s \varnothing t'_{ab})$ underlying flat four-dimensional relative proper metric spacetime $(\mathbb{E}'^3, c_s t')$, valid partially with respect to 3-observers in the relative proper metric Euclidean 3-space and partially with respect to 1-observers in in the relative proper metric time dimension of our universe

Since Fig. 5 contains the metric spacetime and intrinsic metric spacetime dimensions of our universe solely, it can be used to construct absolute intrinsic metric line element, absolute intrinsic metric tensor and absolute intrinsic Ricci tensor (or absolute intrinsic Riemann geometry), on the curved $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$ in our universe, jointly by 3-observers in the relative proper Euclidean 3-space \mathbb{E}'^3 and 1-observers in the relative proper time dimension $c_s t'$ of our universe in it. It is to be recalled from [3] that the 3-observers⁰ in the relative proper Euclidean 3-space \mathbb{E}'^3 of the positive time-universe in Fig. 4 are the ones that appear as 1-observers in the relative proper time dimension $c_s t'$ of our universe, with respect to 3-observers in \mathbb{E}'^3 of our universe in Fig. 5.

On the other hand, by letting, $\mathbb{E}'^3 \rightarrow c_s t^{0'}$, $\varnothing\rho' \rightarrow \varnothing c_s \varnothing t^{0'}$ and $\varnothing\hat{\rho} \rightarrow \varnothing\hat{c}_s \varnothing\hat{t}^0$, in the lower half of Fig. 4, one obtains the diagram in the positive time-universe that corresponds to that of Fig. 5 in the positive (or our) universe. That diagram shall not be drawn here however, since it has no usefulness in our universe.

The projection of the elementary absolute intrinsic metric time coordinate interval $\varnothing\hat{c}_s d\varnothing\hat{t}$ about point P^0 along the curved $\varnothing\hat{c}_s \varnothing\hat{t}$ into the vertical and the projection of the elementary absolute intrinsic metric space interval $d\varnothing\hat{\rho}$ about the symmetry-partner point P along the curved $\varnothing\hat{\rho}$ into the horizontal in Fig. 5, are given respectively as

$$\varnothing c_{sab} d\varnothing t'_{ab} = \varnothing\hat{c}_s d\varnothing\hat{t} \cos \varnothing\hat{\psi}_{P^0}; \quad (\text{w.r.t. 1 - observers in } c_s t') \quad (19a)$$

$$\varnothing\rho'_{ab} = d\varnothing\hat{\rho} \cos \varnothing\hat{\psi}_P = d\varnothing\hat{\rho} \cos \varnothing\hat{\psi}; \quad (\text{w.r.t. 3 - observers in } \mathbb{E}'^3) \quad (19b)$$

The intrinsic metric coordinate interval projection relations (19a) and (19b) derived from Fig. 5 are the modified forms of relations (5a) and (5b) derived from Fig. 4. The expressions (19a) and (19b) can be obtained by simply letting, $d\varnothing\rho'_{ab} \rightarrow \varnothing c_{sab} \varnothing t'_{ab}$ and $d\varnothing\rho^0 \rightarrow \varnothing\hat{c}_s d\varnothing\hat{t}$, in relation (5b), while retaining relation (5a).

Again the 'non-metric' component $\varnothing\hat{c}_s \delta\varnothing\hat{t}$ projected into $\varnothing\rho'_{ab}$ along the horizontal by interval $\varnothing\hat{c}_s d\varnothing\hat{t}$ about point P^0 along the curved $\varnothing\hat{c}_s \varnothing\hat{t}$ and the 'non-metric' component $\delta\varnothing\hat{\rho}$ projected into $\varnothing c_s \varnothing t'_{ab}$ along the vertical by interval $d\varnothing\hat{\rho}$ about the symmetry-partner point P along the curved $\varnothing\hat{\rho}$ in Fig. 5, have not been taken into consideration in the intrinsic metric coordinate interval projection relations (19a) and (19b), since our interest in Eqs. (19a) and (19b) is in deriving the absolute intrinsic metric line element and the implied absolute intrinsic metric tensor, or to construct absolute intrinsic Riemann geometry on the curved 'two-dimensional' absolute intrinsic metric spacetime $(\varnothing\hat{\rho}, \varnothing\hat{c}_s \varnothing\hat{t})$ in Fig. 5, partially with respect to 3-observers in \mathbb{E}'^3 and partially with respect to 1-observers in $c_s t'$ in that figure. The projective 'non-metric' absolute intrinsic coordinate intervals, $\delta\varnothing\hat{\rho}$ and $\varnothing\hat{c}_s \delta\varnothing\hat{t}$, cannot appear in an absolute intrinsic metric line element.

However let us temporarily take into account the projective 'non-metric' components in the intrinsic coordinate projection expressions that can be derived from Fig. 5 to have

$$\varnothing c_{sab} d\varnothing t'_{ab} = \varnothing\hat{c}_s d\varnothing\hat{t} \cos \varnothing\hat{\psi}_{P^0} \quad \text{and} \quad \delta\varnothing\hat{\rho} = d\varnothing\hat{\rho} \sin \varnothing\hat{\psi}_P; \quad (\text{w.r.t. 1 - observers in } c_s t'); \quad (20a)$$

$$d\varnothing\rho'_{ab} = d\varnothing\hat{\rho} \cos \varnothing\hat{\psi}_P \quad \text{and} \quad \varnothing\hat{c}_s \delta\varnothing\hat{t} = \varnothing\hat{c}_s d\varnothing\hat{t} \sin \varnothing\hat{\psi}_{P^0}; \quad (\text{w.r.t. 3 - observers in } \mathbb{E}'^3). \quad (20b)$$

There is invariance of partial intrinsic line element between the curved absolute intrinsic space $\varnothing\hat{\rho}$ and its projective straight line absolute proper intrinsic space $\varnothing\rho'_{ab}$ along the horizontal, with respect to 3-observers in \mathbb{E}'^3 in Fig. 5, expressed as follows

$$(d\varnothing\hat{\rho})^2 = (d\varnothing\rho'_{ab})^2 + \varnothing\hat{c}_s^2 (\delta\varnothing\hat{t})^2,$$

or

$$(d\varnothing\rho'_{ab})^2 = (d\varnothing\hat{\rho})^2 - \varnothing\hat{c}_s^2(\delta\varnothing\hat{t})^2,$$

which upon using system (20b) gives

$$(d\varnothing\rho'_{ab})^2 = (d\varnothing\rho'_{ab})^2 \sec^2 \varnothing\hat{\psi}_P - \varnothing\hat{c}_s^2(d\varnothing\hat{t})^2 \sin^2 \varnothing\hat{\psi}_{P^0} .$$

This simplifies further as follows by virtue of Eq. (19a),

$$(d\varnothing\rho'_{ab})^2 = (d\varnothing\rho'_{ab})^2 \sec^2 \varnothing\hat{\psi}_P - \varnothing c_{sab}^2 (d\varnothing t'_{ab})^2 \tan^2 \varnothing\hat{\psi}_{P^0} ;$$

(w.r.t. 3 – observers in \mathbb{E}'^3) (21a)

There is likewise invariance of partial intrinsic line element between the curved absolute intrinsic time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ and its projective straight line absolute proper intrinsic time dimension $\varnothing c_{sab}\varnothing t'_{ab}$ along the vertical, with respect to 1-observers in $c_s t'$ in Fig. 5, expressed as follows

$$\varnothing\hat{c}_s^2(d\varnothing\hat{t})^2 = \varnothing c_{sab}^2(d\varnothing t'_{ab})^2 + (\delta\varnothing\hat{\rho})^2 ,$$

or

$$\varnothing c_{sab}^2(d\varnothing t'_{ab})^2 = (\varnothing\hat{c}_s^2(d\varnothing\hat{t})^2 - (\delta\varnothing\hat{\rho})^2) ,$$

which upon using system (20a) gives

$$\varnothing c_{sab}^2(d\varnothing t'_{ab})^2 = \varnothing c_{sab}^2(d\varnothing t'_{ab})^2 \sec^2 \varnothing\hat{\psi}_{P^0} - (d\varnothing\hat{\rho})^2 \sin^2 \varnothing\hat{\psi}_P .$$

This simplifies further as follows by virtue of Eq. (19b),

$$\varnothing c_{sab}^2(d\varnothing t'_{ab})^2 = \varnothing c_{sab}^2(d\varnothing t'_{ab})^2 \sec^2 \varnothing\hat{\psi}_{P^0} - (d\varnothing\hat{\rho})^2 \tan^2 \varnothing\hat{\psi}_P ; \quad (\text{w.r.t. 1 – observers in } c_s t') .$$

(21b)

Now the point P along the curved $\varnothing\hat{\rho}$ and the point P^0 along the curved $\varnothing\hat{c}_s\varnothing\hat{t}$ in Fig. 5, are symmetry-partner points. Consequently the absolute intrinsic angles, $\varnothing\hat{\psi}_P$ and $\varnothing\hat{\psi}_{P^0}$, are equal. Thus we shall let, $\varnothing\hat{\psi}_P = \varnothing\hat{\psi}_{P^0} \equiv \varnothing\hat{\psi}$. By using this fact and adding Eqs. (21a) and (21b) we have

$$\varnothing c_{sab}^2(d\varnothing t'_{ab})^2 (d\varnothing\rho'_{ab})^2 = \varnothing c_{sab}^2(d\varnothing t'_{ab})^2 (\sec^2 \varnothing\hat{\psi} - \tan^2 \varnothing\hat{\psi}) + (d\varnothing\rho'_{ab})^2 (\sec^2 \varnothing\hat{\psi} - \tan^2 \varnothing\hat{\psi}) .$$

(22)

Equation (22) expresses intrinsic local Euclidean invariance ($\varnothing\text{LEI}$) in terms of absolute proper intrinsic coordinate intervals, partially with respect to 3-observers in \mathbb{E}'^3 and partially with respect to 1-observers in $c_s t'$ in Fig. 5, by virtue of the expression, $\sec^2 \varnothing\hat{\psi} - \tan^2 \varnothing\hat{\psi} = 1$. The full invariance of intrinsic Euclidean line element (22) has been written partially as Eq. (21a) with respect to 3-observers in \mathbb{E}'^3 from the lower half of Fig. 5, and partially as Eq. (21b) with respect to 1-observers in $c_s t'$ from the upper half of Fig. 5.

The invariance of intrinsic line element between the curved absolute intrinsic metric spacetime ($\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t}$) and its projective flat absolute proper intrinsic metric spacetime ($\varnothing\rho'_{ab}, \varnothing c_{sab}\varnothing t'_{ab}$) in Fig. 5 is expressed formally as

$$\begin{aligned} (d\varnothing\hat{s}^*)^2 &= (d\varnothing s'^*)^2 \\ \varnothing\hat{c}_s^2(d\varnothing\hat{t})^2 + (d\varnothing\hat{\rho})^2 &= \varnothing c_s^2(d\varnothing t'_{ab})^2 + (d\varnothing\rho'_{ab})^2 \end{aligned}$$

(23)

This is simply Eq. (9) with $d\varnothing\hat{\rho}^0$ replaced by $\varnothing\hat{c}_s\varnothing\hat{t}$ and $\varnothing\rho'^0_{ab}$ replaced by $\varnothing c_{sab}d\varnothing t'_{ab}$.

It follows from Eq. (23) that the absolute proper intrinsic metric space interval $d\varnothing\rho'_{ab}$ and the absolute proper intrinsic metric time dimension interval $\varnothing c_{sab}d\varnothing t'_{ab}$ can be replaced with the absolute intrinsic

metric space interval $d\varnothing\hat{\rho}$ and the absolute intrinsic metric time 'dimension' interval $\varnothing\hat{c}_s d\varnothing\hat{t}$ respectively in Eq. (22) to have

$$\begin{aligned} \varnothing\hat{c}_s^2(d\varnothing\hat{t})^2(d\varnothing\hat{\rho})^2 &= \varnothing\hat{c}_s^2(d\varnothing\hat{t})^2 \left(\sec^2 \varnothing\hat{\psi} - \tan^2 \varnothing\hat{\psi} \right) \\ &+ (d\varnothing\hat{\rho})^2 \left(\sec^2 \varnothing\hat{\psi} - \tan^2 \varnothing\hat{\psi} \right) . \end{aligned} \quad (24)$$

Equation (24) expresses absolute intrinsic local Euclidean invariance (A \varnothing LEI) in terms of absolute intrinsic coordinate intervals, partially with respect to 3-observers in \mathbb{E}'^3 and partially with respect to 1-observers in $c_s t'$ in Fig. 5. Let us replace the left-hand side of Eq. (24) by the square of absolute intrinsic line element to still be denoted by $(d\varnothing\hat{s}^*)^2$ to have

$$\begin{aligned} (d\varnothing\hat{s}^*)^2 &= \varnothing\hat{c}_s^2(d\varnothing\hat{t})^2(\sec^2 \varnothing\hat{\psi} - \tan^2 \varnothing\hat{\psi}) + \\ &(d\varnothing\hat{\rho})^2(\sec^2 \varnothing\hat{\psi} - \tan^2 \varnothing\hat{\psi}) \end{aligned} \quad (25)$$

or

$$(d\varnothing\hat{s}^*)^2 = \varnothing\hat{c}_s^2(d\varnothing\hat{t})^2 + (d\varnothing\hat{\rho})^2 . \quad (26)$$

The star label introduced on the absolute intrinsic line element on the curved 'two-dimensional' absolute intrinsic metric space $(\varnothing\hat{\rho}, \varnothing\hat{\rho}^0)$ in Fig. 4, is retained on the curved absolute intrinsic metric spacetime $(\varnothing\hat{\rho}, \varnothing\hat{c}_s \varnothing\hat{t})$ in Fig. 5, because Fig. 5 is the same as Fig. 4. Figure 5 has been obtained by simply transforming the space coordinates and intrinsic space coordinates of the positive time-universe into the time and intrinsic time coordinates of our universe in the upper half of Fig. 4. The absolute intrinsic line element and absolute intrinsic metric tensor derived partially with respect to 3-observers in \mathbb{E}'^3 and partially with respect to 3-observers in $\mathbb{E}^{0/3}$ in Fig. 4, remain unchanged partially with respect to 3-observers in \mathbb{E}'^3 and partially with respect to 1-observers in $c_s t'$ in Fig. 5.

The absolute intrinsic Euclidean line element (25) or (26) obtains at every point along the curved $\varnothing\hat{\rho}$ and the symmetry-partner point along the curved $\varnothing\hat{c}_s \varnothing\hat{t}$, with respect to 3-observers in \mathbb{E}'^3 and 1-observers in $c_s t'$ in Fig. 5. This is so in so far as both the metric and 'non-metric' intrinsic coordinate interval projections are taken into account in deriving intrinsic coordinate projection relations from Fig. 5, as done in systems (20a) and (20b) and in Eqs. (21a) and (21b).

As done with $(d\varnothing\hat{s}^*)^2$ in Eq. (11) earlier, let us separate the absolute intrinsic Euclidean line element in Eq. (25) into the metric and 'non-metric' components $(d\varnothing\hat{s}_m^*)^2$ and $(d\varnothing\hat{s}_{nm}^*)^2$ as

$$\begin{aligned} (d\varnothing\hat{s}^*)^2 &= (d\varnothing\hat{s}_m^*)^2 + (d\varnothing\hat{s}_{nm}^*)^2 \\ &= \sum_{i,j=0}^1 \varnothing\hat{g}_{ij}^* d\varnothing\hat{x}^i d\varnothing\hat{x}^j = \sum_{i,j=0}^1 \varnothing\hat{R}_{ij}^* d\varnothing\hat{x}^i d\varnothing\hat{x}^j \end{aligned} \quad (27)$$

or

$$\begin{aligned} (d\varnothing\hat{s}^*)^2 &= \left(\varnothing\hat{c}_s^2 d\varnothing\hat{t}^2 \sec^2 \varnothing\hat{\psi} + d\varnothing\hat{\rho}^2 \sec^2 \varnothing\hat{\psi} \right) \\ &- \left(\varnothing\hat{c}_s^2 d\varnothing\hat{t}^2 \tan^2 \varnothing\hat{\psi} + \tan^2 \varnothing\hat{\psi} d\varnothing\hat{\rho}^2 \right) . \end{aligned} \quad (28)$$

The absolute intrinsic metric line element on the curved 'two-dimensional' absolute intrinsic metric spacetime $(\varnothing\hat{\rho}, \varnothing\hat{c}_s \varnothing\hat{t})$, which is valid partially with respect to 3-observers in \mathbb{E}'^3 and partially with respect to 1-observers in $c_s t'$ in Fig. 5 that follows from Eqs. (27) and (28), is the following

$$\begin{aligned} (d\varnothing s_m^*)^2 &= \sum_{i,j=0}^1 \varnothing \hat{g}_{ij}^* d\varnothing \hat{x}^i d\varnothing \hat{x}^j ; \\ &= \varnothing \hat{g}_{00}^* \varnothing \hat{c}_s^2 d\varnothing \hat{t}^2 + \varnothing \hat{g}_{11}^* d\varnothing \hat{\rho}^2 ; \end{aligned} \quad (29)$$

$$= \sec^2 \varnothing \hat{\psi} \varnothing \hat{c}_s^2 d\varnothing \hat{t}^2 + \sec^2 \varnothing \hat{\psi} d\varnothing \hat{\rho}^2 ; \quad (30)$$

$$= \frac{\varnothing \hat{c}_s^2 d\varnothing \hat{t}^2}{1 - \varnothing \hat{k}^2} + \frac{d\varnothing \hat{\rho}^2}{1 - \varnothing \hat{k}^2} . \quad (31)$$

where the relation, $\varnothing \hat{k} = \sin \varnothing \hat{\psi}$, derived in sub-section 1.1 of [2] and presented as Eq. (13) of that article has been used.

The absolute intrinsic metric tensor implied by the absolute intrinsic line element (30) or (31) is

$$\varnothing \hat{g}_{ij}^* = \begin{pmatrix} \sec^2 \varnothing \hat{\psi} & 0 \\ 0 & \sec^2 \varnothing \hat{\psi} \end{pmatrix} \quad (32)$$

or

$$\varnothing \hat{g}_{ij}^* = \begin{pmatrix} \frac{1}{1 - \varnothing \hat{k}^2} & 0 \\ 0 & \frac{1}{1 - \varnothing \hat{k}^2} \end{pmatrix} . \quad (33)$$

Again, the component,

$$\varnothing \hat{g}_{00}^* \varnothing \hat{c}_s^2 d\varnothing \hat{t}^2 = \varnothing \hat{c}_s^2 d\varnothing \hat{t}^2 / (1 - \varnothing \hat{k}^2) ,$$

in the absolute intrinsic line element (29), (30) or (31) has been derived by and is hence valid with respect to 1-observers in the relative proper time dimension $c_s t'$, while the component,

$$\varnothing \hat{g}_{11}^* d\varnothing \hat{\rho}^2 = d\varnothing \hat{\rho}^2 \sec^2 \varnothing \hat{\psi} = d\varnothing \hat{\rho}^2 / (1 - \varnothing \hat{k}^2) ,$$

has been derived by, and is hence valid with respect to 3-observers in the relative proper Euclidean 3-space \mathbb{E}^3 in Fig. 5.

Now the absolute intrinsic line element (16) or (17) on the curved 'two-dimensional' absolute intrinsic metric space $(\varnothing \hat{\rho}, \varnothing \hat{\rho}^0)$ in Fig. 4, obtained by uniting Fig. 2 (with respect to 3-observers in \mathbb{E}^3) and Fig. 3 (with respect to 3-observers⁰ in $\mathbb{E}^{0/3}$), possesses the circular structure like the absolute intrinsic line element on the curved 'two-dimensional' and 'three-dimensional' absolute intrinsic metric spaces $\varnothing \hat{\mathbb{M}}^2$ or $\varnothing \hat{\mathbb{M}}^3$ encountered in part two of this paper [2]; compare the absolute intrinsic line element (16) or (17) on the curved $(\varnothing \hat{\rho}, \varnothing \hat{\rho}^0)$ in Fig. 4 above with the absolute intrinsic line elements (2d) and (3) on $\varnothing \hat{\mathbb{M}}^2$ and $\varnothing \hat{\mathbb{M}}^3$ in [2]. The absolute intrinsic line element (30) or (31) on the curved 'two-dimensional' absolute intrinsic metric spacetime $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$ in Fig. 5, which is valid partially with respect to 3-observers in \mathbb{E}^3 and partially with respect to 1-observers in $c_s t'$ in that figure, likewise possesses the circular structure like the absolute intrinsic line element (2d) on curved 'two-dimensional' absolute intrinsic metric space $\varnothing \hat{\mathbb{M}}^2$ in [2].

It follows from the preceding paragraph that the pair of absolute intrinsic tensor equations derived for curved 'two-dimensional' and 'three-dimensional' absolute intrinsic metric spaces $\varnothing \hat{\mathbb{M}}^2$ and $\varnothing \hat{\mathbb{M}}^3$ in [2] and presented as Eqs.(33) and (45) of that article, are equally valid for the curved 'two-dimensional' absolute intrinsic metric space $(\varnothing \hat{\rho}, \varnothing \hat{\rho}^0)$ in Fig. 4 and the curved 'two-dimensional' absolute intrinsic metric spacetime $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$ in Fig. 5. Let us then write those absolute intrinsic tensor equations in terms of starred absolute intrinsic metric tensor and starred absolute intrinsic Ricci tensor on the curved 'two-dimensional' absolute intrinsic spacetime $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$ in Fig. 5 as follows

$$\varnothing \hat{g}_{ij}^* - \varnothing \hat{R}_{ij}^* = \delta_{ij} \text{ (A}\varnothing\text{LEI)} . \quad (34)$$

For the second absolute intrinsic tensor equation, let us start with the intermediate equation (42) of [2] in the process of derivation of that equation in that article namely,

$$\varnothing \hat{R}^*_{ij} - \frac{\text{Tr } \varnothing \hat{R}^{*i}_i}{n} \varnothing \hat{g}^*_{ij} = 0 . \quad (35)$$

where n is the dimensionality of the absolute intrinsic metric space and of the matrix $\varnothing \hat{R}^{*i}_i$. For the curved $(\varnothing \hat{\rho}, \varnothing \hat{\rho}^0)$ in Fig. 4, which is replaced by the curved $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$ in Fig. 4, being considered here, $n = 2$, thereby simplifying (35) as

$$\varnothing \hat{R}^*_{ij} - \frac{1}{2} \varnothing \hat{R}^* \varnothing \hat{g}^*_{ij} = 0 , \quad (36)$$

where the absolute intrinsic Riemann scalar $\varnothing \hat{R}^*$ is the trace of the 2×2 diagonal matrix $\varnothing \hat{R}^{*i}_i$.

Interestingly Eq. (36) in absolute intrinsic Riemann geometry on curved 'two-dimensional' absolute intrinsic metric spacetimes $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$ in Fig. 5, takes on its form in the context of conventional Riemann geometry namely,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 0 . \quad (37)$$

However the factor $\frac{1}{2}$ in the term $-\frac{1}{2} \varnothing \hat{R}^* \varnothing \hat{g}^*_{ij}$ in (36) restricts absolute intrinsic metric spaces to curved 'two-dimensional' absolute intrinsic metric spaces of the type $(\varnothing \hat{\rho}, \varnothing \hat{\rho}^0)$ in Fig. 4, which must be replaced by the curved 'two-dimensional' absolute intrinsic metric spacetime $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$ in Fig. 5, while the factor $\frac{1}{2}$ in the term $-\frac{1}{2} R g_{\mu\nu}$ in (37) in conventional Riemann geometry is not known to restrict the dimensionality of a conventional Riemann space, or conventional Riemann spacetime to 2. A conventional Riemann space M^p can be of any dimension p and a conventional Riemannian spacetime M^{p+q} can be of any dimension $p + q$; for instance, $p = 3, q = 1$, in the case of the curved four-dimensional spacetime of the general theory of relativity. Equation (37) is known to apply to all conventional Riemann spaces and conventional Riemannian spacetimes without restriction on their dimensionality.

As derived and presented as Eq. (45) in [2], Eq. (36) admits of further simplification as

$$\varnothing \hat{R}^*_{ij} - \varnothing \hat{k}^2 \varnothing \hat{g}^*_{ij} = 0 , \quad (38)$$

where $\varnothing \hat{k}$ is the equal absolute intrinsic curvature parameter of an arbitrary point along the curved $\varnothing \hat{\rho}$ and its symmetry-partner point along the curved $\varnothing \hat{\rho}^0$ in Fig. 4, which become an arbitrary point along the curved $\varnothing \hat{\rho}$ and its symmetry-partner point along the curved $\varnothing \hat{c}_s \varnothing \hat{t}$ in Fig. 5.

The perfect symmetry of state between our (or positive) universe and the positive time-universe makes absolute intrinsic curvature parameters, $\varnothing \hat{k}_P$ and $\varnothing \hat{k}_{P0}$, at every pair of symmetry-partner points along the curved $\varnothing \hat{\rho}$ and $\varnothing \hat{c}_s \varnothing \hat{t}$ respectively in Fig. 5 to be identical; that is, $\varnothing \hat{k}_P = \varnothing \hat{k}_{P0} \equiv \varnothing \hat{k}$. It is the square of the identical absolute intrinsic curvature parameters $\varnothing \hat{k}^2$ that appears as the diagonal entries of the 2×2 diagonal matrix $\varnothing \hat{R}^{*i}_i$ in Eq. (35), for which $n = 2$. Hence, $\text{Tr } \varnothing \hat{R}^{*i}_i = \varnothing \hat{R}^* = 2\varnothing \hat{k}^2$ and $\frac{1}{2} \varnothing \hat{R}^* = \varnothing \hat{k}^2$, which makes Eq. (38) the same as Eq. (36).

It is the pair of absolute intrinsic tensor equations (34) and (38), written as Eqs. (33) and (45) of [2] (and not (34) and (36) above), that shall be found directly applicable in absolute intrinsic Riemann geometry on curved 'two-dimensional' absolute intrinsic metric spacetime $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$, partially with respect to 3-observers in \mathbb{E}^{13} and partially with respect to 1-observers in $c_s t'$ in Fig. 5. For instance, the (algebraic) solution to Eqs. (34) and (38) are the starred absolute intrinsic metric tensor (33) and the following starred absolute intrinsic Ricci tensor,

$$\varnothing \hat{R}^*_{ij} = \begin{pmatrix} \frac{\varnothing \hat{k}^2}{1 - \varnothing \hat{k}^2} & 0 \\ 0 & \frac{\varnothing \hat{k}^2}{1 - \varnothing \hat{k}^2} \end{pmatrix} . \quad (39)$$

Now let us consider the superposition of a pair of ‘2-dimensional’ absolute intrinsic metric spacetimes (a pair of ‘2-dimensional’ absolute intrinsic Riemannian metric spacetimes) $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$ and $(\varnothing\hat{\rho}', \varnothing\hat{c}_s\varnothing\hat{t}')$, such that $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$ lies over (or is curved relative to) $(\varnothing\hat{\rho}', \varnothing\hat{c}_s\varnothing\hat{t}')$, as illustrated in Fig. 6.

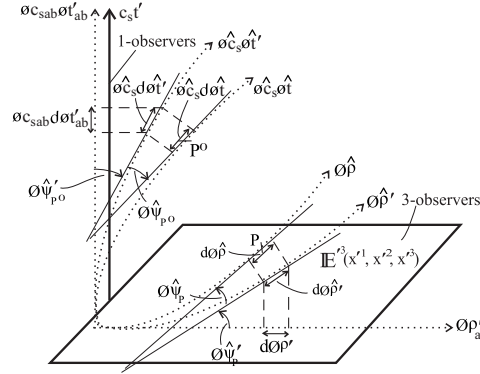


Fig. 6. A pair of co-existing ‘two-dimensional’ absolute intrinsic metric spacetimes and their underlying flat two-dimensional absolute proper intrinsic metric spacetime underlying the flat four-dimensional relative proper metric spacetime; the lower half of this figure is valid with respect to 3-observers in the relative proper Euclidean 3-space and the upper half is valid with respect to 1-observers in the relative proper time dimension

The pair of absolute intrinsic tensor equations (34) and (38) must be written in terms of resultant starred absolute intrinsic metric tensor and resultant starred absolute intrinsic Ricci tensor as

$$\varnothing\hat{g}^*_{ij} - \varnothing\hat{R}^*_{ij} = \delta_{ij} ; \quad (40)$$

$$\varnothing\hat{R}^*_{ij} - \varnothing\hat{k}^2 \varnothing\hat{g}^*_{ij} = 0 , \quad (41)$$

where the resultant absolute intrinsic curvature parameter $\varnothing\hat{k}$ for the purpose of writing the resultant absolute intrinsic line element and resultant absolute intrinsic metric tensor at an arbitrary point of the upper curved absolute intrinsic metric space $\varnothing\hat{\rho}$ relative to $\varnothing\rho'_{ab}$ along the horizontal, or at the symmetry-partner point of the upper curved absolute intrinsic metric time ‘dimension’ $\varnothing\hat{c}_s\varnothing\hat{t}$ relative to $\varnothing c_{sab}\varnothing t'_{ab}$ along the vertical, is given in terms of the individual absolute intrinsic curvature parameters $\varnothing\hat{k}'$ at point P' of the lower curved absolute intrinsic metric space $\varnothing\hat{\rho}'$ and $\varnothing\hat{k}$ at point P of the upper curved absolute intrinsic metric space $\varnothing\hat{\rho}$, prior to their superposition as follows, as derived in sub-section 1.6 of part one of this paper [2]

$$\varnothing\hat{k}^2 = (\varnothing\hat{k}')^2 + \varnothing\hat{k}^2 . \quad (42)$$

The resultant starred absolute intrinsic tensors $\varnothing\hat{g}^*_{ij}$ and $\varnothing\hat{R}^*_{ij}$ that satisfy equations (40) and (41) are the following

$$\begin{aligned} \varnothing\hat{g}^*_{ij} &= \begin{pmatrix} \frac{1}{1 - \varnothing\hat{k}^2} & 0 \\ 0 & \frac{1}{1 - \varnothing\hat{k}^2} \end{pmatrix} ; \\ &= \begin{pmatrix} \frac{1}{1 - (\varnothing\hat{k}')^2 - \varnothing\hat{k}^2} & 0 \\ 0 & \frac{1}{1 - (\varnothing\hat{k}')^2 - \varnothing\hat{k}^2} \end{pmatrix} \end{aligned} \quad (43)$$

and

$$\begin{aligned} \varnothing \hat{R}_{ij}^* &= \begin{pmatrix} \frac{\hat{\varnothing k}^2}{1 - \hat{\varnothing k}^2} & 0 \\ 0 & \frac{\hat{\varnothing k}^2}{1 - \hat{\varnothing k}^2} \end{pmatrix}; \\ &= \begin{pmatrix} \frac{(\hat{\varnothing k}')^2 + \hat{\varnothing k}^2}{1 - (\hat{\varnothing k}')^2 - \hat{\varnothing k}^2} & 0 \\ 0 & \frac{(\hat{\varnothing k}')^2 + \hat{\varnothing k}^2}{1 - (\hat{\varnothing k}')^2 - \hat{\varnothing k}^2} \end{pmatrix}. \end{aligned} \tag{44}$$

The resultant absolute intrinsic line element on the upper curved ‘two-dimensional’ absolute intrinsic spacetime with curved absolute intrinsic ‘dimensions’ $\varnothing \hat{\rho}$ and $\varnothing \hat{c}_s \varnothing \hat{t}$ in Fig. 6, is then given partially with respect to 3-observers in \mathbb{IE}^3 and partially with respect to 1-observers in $c_s t'$ (or with respect to (3+1)-observers in $(\mathbb{IE}^3, c_s t')$) as

$$\begin{aligned} (d\varnothing \hat{s}^*)^2 &= \varnothing \hat{g}_{00}^* \varnothing \hat{c}_s^2 d\varnothing \hat{t}^2 + \varnothing \hat{g}_{11}^* d\varnothing \hat{\rho}^2 \\ &= \frac{\varnothing \hat{c}_s^2 d\varnothing \hat{t}^2}{1 - (\hat{\varnothing k}')^2 - \hat{\varnothing k}^2} + \frac{d\varnothing \hat{\rho}^2}{1 - (\hat{\varnothing k}')^2 - \hat{\varnothing k}^2}. \end{aligned} \tag{45}$$

On the other hand, the projection of the elementary coordinate interval $d\varnothing \hat{\rho}$ about point P of the upper curved absolute intrinsic space $\varnothing \hat{\rho}$ into absolute proper intrinsic space $\varnothing \rho'_{ab}$ along the horizontal and of interval $\varnothing \hat{c}_s d\varnothing \hat{t}$ about point P⁰ of the upper curved absolute intrinsic time ‘dimension’ $\varnothing \hat{c}_s \varnothing \hat{t}$ into the absolute proper intrinsic time dimension $\varnothing c_{sab} \varnothing t'_{ab}$ along the vertical, are given in terms of the resultant absolute intrinsic angle, $\varnothing \hat{\psi}_{res} = \varnothing \hat{\psi} + \varnothing \hat{\psi}'$, as follows, as derived in sub-sub-section 1.6.2 of part one of this paper [2],

$$\begin{aligned} d\varnothing \rho'_{ab} &= d\varnothing \hat{\rho} \cos \varnothing \hat{\psi}_{res} \\ &= d\varnothing \hat{\rho} \cos(\varnothing \hat{\psi} + \varnothing \hat{\psi}') \\ &= d\varnothing \hat{\rho} \cos \varnothing \hat{\psi}' \cos \varnothing \hat{\psi} \\ &= d\varnothing \hat{\rho} (1 - \hat{\varnothing k}'^2)^{1/2} (1 - \hat{\varnothing k}^2)^{1/2}; \\ &\quad (\text{w.r.t. 3 - observers in } \mathbb{IE}^3); \\ \varnothing c_{sab} d\varnothing t'_{ab} &= \varnothing \hat{c}_s d\varnothing \hat{t} \cos \varnothing \hat{\psi}_{res} \\ &= d\varnothing \hat{\rho} \cos(\varnothing \hat{\psi} + \varnothing \hat{\psi}') \\ &= \varnothing \hat{c}_s d\varnothing \hat{t} \cos \varnothing \hat{\psi}' \cos \varnothing \hat{\psi} \\ &= \varnothing \hat{c}_s d\varnothing \hat{t} (1 - \hat{\varnothing k}'^2)^{1/2} (1 - \hat{\varnothing k}^2)^{1/2}; \\ &\quad (\text{w.r.t. 1 - observers in } c_s t'). \end{aligned} \tag{46}$$

Extension of relations (42) through Eq. (47) to a situation of the superposition of three and larger number of curved ‘two-dimensional’ absolute intrinsic metric spacetimes is easy and straight forward.

It is at the first step of the modification of Fig. 4 to the form in which it is valid for absolute intrinsic Riemann geometry in our universe, when Fig. 4 is converted to Fig. 5, that the starred absolute intrinsic tensor equations (34) and (38) must be solved to obtain the starred absolute intrinsic metric tensor (33) and starred absolute intrinsic Ricci tensor (39). The starred absolute intrinsic metric tensor (33), the starred absolute intrinsic Ricci tensor (39) and Fig. 5 they are associated

with, all of which are valid partially with respect to 3-observers in the proper metric Euclidean 3-space \mathbb{E}^3 and partially with respect to 1-observers in the proper metric time dimension $c_s t'$ in Fig. 5, shall now be modified to the forms in which they are valid with respect to 3-observers in the proper Euclidean 3-space \mathbb{E}^3 solely. This will be at the second (and final) step of converting Fig. 4 and the associated absolute intrinsic line element (30) or (31), the implied absolute intrinsic metric tensor (33) and absolute intrinsic Ricci tensor (39), to the forms in which they are valid with respect to 3-observers in the relative proper Euclidean 3-space \mathbb{E}^3 solely. The modified form of Fig. 5 to be derived is the valid diagram and the associated modified absolute intrinsic line element, absolute intrinsic metric tensor and absolute intrinsic Ricci tensor, are the valid forms in the context of absolute intrinsic Riemannian spacetime geometry in our universe.

2.1 The Form of Spacetime/Intrinsic Spacetime Diagram of Absolute Intrinsic Riemannian Metric Spacetime Geometry that is Valid with Respect to 3-observers in the Relative Proper Euclidean 3-space Solely

Now the starred absolute intrinsic line element (30) or (31) and the implied starred absolute intrinsic metric tensor (32) or (33), although have been derived on the curved 'two-dimensional' absolute intrinsic metric spacetime (or 'two-dimensional' absolute metric nospacetime-notime) $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$ in Fig.5, do not possess the hyperbolic structure of the metric tensors on Riemannian metric spacetime manifolds of the type, $M^{p+q}; p = 3, q = 1$. Rather they have the circular/elliptical structure of the metric tensors of Riemannian metric spaces without time dimension of the class M^p . The fact that the proper time dimension $c_s t'$, the absolute proper intrinsic time dimension $\varnothing c_{sab}\varnothing t'_{ab}$ and the curved absolute intrinsic time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$, appear in Fig. 5 (to replace $\mathbb{E}^{0,3}$, $\varnothing\rho^{0'}_{ab}$ and $\varnothing\hat{\rho}^0$ respectively in Fig. 4), has not shown up in the structure of the absolute intrinsic line element

(30) or (31) and the implied absolute intrinsic metric tensor (32) or (33) on the curved 'two-dimensional' absolute intrinsic metric spacetime $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$ in Fig. 5.

The circular structure of the absolute intrinsic line element (30) or (31) and of the absolute intrinsic metric tensor (32) or (33) arises, because they have been derived partially from the upper half of Fig. 5 by or with respect to 1-observers in the relative proper time dimension $c_s t'$ along the vertical and partially from the lower half of that figure by or with respect to 3-observers in the relative proper Euclidean 3-space \mathbb{E}^3 along the horizontal. The existence of the curved absolute intrinsic time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ does not appear in physics formulated in the lower half by 3-observers in \mathbb{E}^3 , just as the curved absolute intrinsic space $\varnothing\hat{\rho}$ does not appear in physics formulated in the upper half by 1-observers in $c_s t'$ in Fig. 5. Consequently the absolute intrinsic line element (30) or (31) obtained by uniting the partial absolute intrinsic line element (3) or (4) derived on the curved $\varnothing\hat{c}_s\varnothing\hat{t}$ by 1-observers in $c_s t'$ and the partial absolute intrinsic line element (1) or (2) derived on the curved $\varnothing\hat{\rho}$ by 3-observers in \mathbb{E}^3 in Fig. 5, has not assumed the hyperbolic structure expected on a curved 'two-dimensional' absolute intrinsic spacetime.

The purpose of this sub-section is to derive the form of Fig. 5 that is valid with respect to 3-observers in the relative proper physical Euclidean 3-space \mathbb{E}^3 solely in that figure and to derive the corresponding modified forms of the starred absolute intrinsic line element (30) or (31), the starred absolute intrinsic metric tensor (32) or (33) and the starred absolute intrinsic Ricci tensor (39) from the modified diagram. It is the modified diagram and the associated modified forms of the absolute intrinsic line element and absolute intrinsic metric tensor and modified absolute intrinsic Ricci tensor that are valid for absolute intrinsic Riemann geometry in our universe, as shall be justified.

Now let us present the reference geometry to the geometry of Fig. 5 as Fig. 7. Figure 7 will exist in the absence of absolute intrinsic Riemann geometry, thereby making the curved absolute intrinsic metric space $\varnothing\hat{\rho}$ and curved absolute

intrinsic metric time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ in Fig. 5 to become the extended straight line absolute intrinsic metric space $\varnothing\hat{\rho}$ along the horizontal and the extended straight line absolute intrinsic metric time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ along the vertical respectively as in Fig. 7.

The reference geometry to absolute intrinsic Riemannian spacetime geometry of Fig. 7 will persist in the absence of a long-range metric force field. However let us introduce the source of a long-range absolute metric force field at a point S on the flat absolute metric space $\hat{\mathbb{E}}^3$. Then its underlying source of long-range absolute intrinsic metric force field will appear automatically in the underlying straight line absolute intrinsic metric space $\varnothing\hat{\rho}$ directly underneath the source of absolute metric force field introduced at point S in $\hat{\mathbb{E}}^3$. When we particularize to gravitational field, as shall be done fully elsewhere, this means that the absolute rest mass \hat{M}_0 of a gravitational field source is introduced at point S in $\hat{\mathbb{E}}^3$ and the absolute intrinsic rest mass $\varnothing\hat{M}_0$ of the gravitational field source appears automatically in $\varnothing\hat{\rho}$ underneath \hat{M}_0 in $\hat{\mathbb{E}}^3$. As shall be explained to some extent in the last section of this article and completely elsewhere with further development, this action will cause Fig. 6 to evolve into Fig. 5.

Now the absolute metric time 'dimension' $\hat{c}_s\hat{t}$ and the absolute intrinsic metric time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ remain unchanged, that is, do not transform into absolute proper metric time dimension $c_{sab}t'_{ab}$ and absolute proper intrinsic metric time dimension $\varnothing c_{sab}\varnothing t'_{ab}$ respectively in absolute physics/absolute intrinsic physics, with respect to 3-observers in the relative proper Euclidean 3-space \mathbb{E}'^3 , such as associated with the presence of absolute metric force field in absolute spacetime and absolute intrinsic metric force field in absolute intrinsic spacetime, which causes Fig. 7 to transform into Fig. 5 discussed above. Graphically, this means that $\hat{c}_s\hat{t}$ and $\varnothing\hat{c}_s\varnothing\hat{t}$ along the vertical in Fig. 7 will remain along the vertical, with respect to 3-observers in the relative proper Euclidean 3-space \mathbb{E}'^3 , as happens in Fig. 2, which is valid with respect to 3-observers in \mathbb{E}'^3 solely, in the context of absolute physics/absolute intrinsic physics. The absolute time 'dimension' $\hat{c}_s\hat{t}$ and the absolute intrinsic time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ must likewise

remain along the vertical in the modified form of Fig. 5 being sought, which is valid with respect to 3-observers in the relative proper Euclidean 3-space \mathbb{E}'^3 solely, in the context of absolute physics/absolute intrinsic physics.

Thus for the purposes of deriving absolute intrinsic line element and its implied absolute intrinsic metric tensor on the curved 'two-dimensional' absolute intrinsic metric spacetime ($\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t}$) and formulating the non-detectable absolute metric theories of physics as 3-geometry theories in the relative proper Euclidean 3-space - absolute time parameter ($\mathbb{E}'^3; \hat{t}$) and absolute intrinsic metric theories of physics as intrinsic 1-geometry theories in the underlying relative proper intrinsic space - absolute intrinsic time parameter ($\varnothing\rho'; \varnothing\hat{t}$), with respect to 3-observers in \mathbb{E}'^3 solely, the preceding paragraph makes it mandatory for us to modify Fig. 5 in such a way that the absolute time 'dimension' $\hat{c}_s\hat{t}$ and the absolute intrinsic time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ remain along the vertical in the modified diagram.

Now the anti-clockwise sense of rotation by positive absolute intrinsic angle $\varnothing\hat{\psi}_P$ of the absolute intrinsic space interval $d\varnothing\hat{\rho}$ relative to its projective absolute proper intrinsic space interval $d\varnothing\rho'_{ab}$ along the horizontal, is valid with respect to 3-observers in the relative proper physical Euclidean 3-space \mathbb{E}'^3 in Fig. 5. This is so because anti-clockwise rotation is positive with respect to these observers. Likewise the clockwise rotation by positive absolute intrinsic angle $\varnothing\hat{\psi}_{P0}$ of the absolute intrinsic time coordinate interval $\varnothing\hat{c}_s d\varnothing\hat{t}$ relative to its projective absolute proper intrinsic time coordinate interval $\varnothing c_{sab} d\varnothing t'_{ab}$ along the vertical, is valid with respect to 1-observers in the relative proper metric time dimension $c_s t'$ in Fig. 5. This is so because clockwise rotation is positive with respect to these observers (as explained in detail in section 4 of [3]). On the other hand, the clockwise rotation by positive absolute intrinsic angle $\varnothing\hat{\psi}_{P0}$ of the absolute intrinsic time coordinate interval $\varnothing\hat{c}_s d\varnothing\hat{t}$ relative to $\varnothing c_{sab} d\varnothing t'_{ab}$ along the vertical is not valid with respect to 3-observers in 3-space \mathbb{E}'^3 . Consequently the upper half of Fig. 5 is valid with respect to 1-observers in $c_s t'$, while the lower half is valid with respect to 3-observers in \mathbb{E}'^3 .

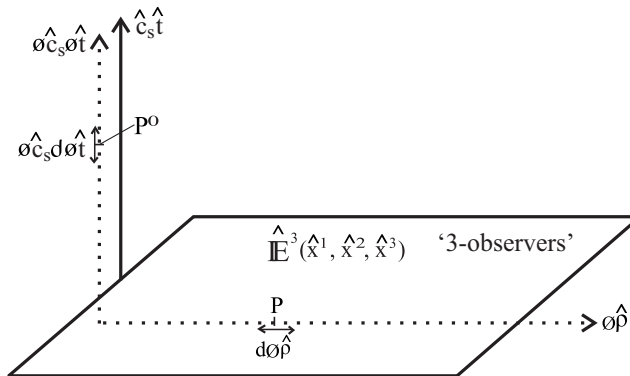


Fig. 7. Flat 'four-dimensional' absolute metric spacetime and its underlying flat 'two-dimensional' absolute intrinsic metric spacetime with the assumed absence of a long-range metric force field (or absence of absolute intrinsic Riemannian spacetime geometry)

In order to make Fig. 5 valid with respect to 3-observers in 3-space \mathbb{E}'^3 solely, as required in the 3-geometry/intrinsic 1-geometry absolute intrinsic Riemann geometry, we must change the positive sign of the absolute intrinsic angle $\hat{\psi}_{P^0}$ of inclination of $\hat{\partial} \hat{c}_s d\hat{t}$ to $\hat{\partial} c_{sab} d\hat{t}'_{ab}$ without changing its clockwise sense. However we can do this only if we also interchange the interval $\hat{\partial} \hat{c}_s d\hat{t}$ along the curved $\hat{\partial} \hat{c}_s \hat{\partial} \hat{t}$ and its projection $\hat{\partial} c_{sab} d\hat{t}'_{ab}$ along the vertical. Doing this about every point along the curved $\hat{\partial} \hat{c}_s \hat{\partial} \hat{t}$ implies

interchanging the curved $\hat{\partial} \hat{c}_s \hat{\partial} \hat{t}$ and the straight line absolute proper intrinsic time dimension $\hat{\partial} c_{sab} \hat{\partial} t'_{ab}$. By implementing these in Fig. 4 we have Fig. 8. The entire Fig. 8 is valid with respect to 3-observers in the relative physical proper Euclidean 3-space \mathbb{E}'^3 solely. This is so because negative angle of clockwise rotation of the curved $\hat{\partial} c_{sab} \hat{\partial} t'_{ab}$ relative to the straight line $\hat{\partial} \hat{c}_s \hat{\partial} \hat{t}$ in Fig. 8 is equivalent to anticlockwise rotation by positive angle, which is valid with respect to 3-observers in \mathbb{E}'^3 .

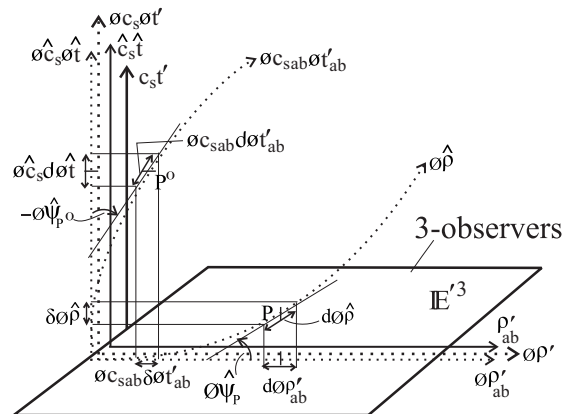


Fig. 8. The form of Fig. 5 that is valid with respect to 3-observers in the relative proper physical Euclidean 3-space solely; the correct diagram for absolute intrinsic Riemannian spacetime geometry in our universe.

It is to be observed that the straight line absolute intrinsic metric time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ and its outward manifestation namely, the absolute metric time 'dimension' $\hat{c}_s\hat{t}$, exist as straight line 'dimensions' along the vertical with respect to 3-observers in the relative proper metric Euclidean 3-space \mathbb{E}'^3 in the correct absolute intrinsic Riemannian metric spacetime geometry of Fig. 8. The relative proper intrinsic metric time dimension $\varnothing c_s \varnothing t'$ and its outward manifestation namely, the relative proper metric time dimension $c_s t'$ also appear along the vertical in Fig. 8. This is possible because $\varnothing c_s \varnothing t'$ and $c_s t'$ are not intrinsic dimensions and dimension of absolute intrinsic Riemannian spacetime geometry and since they actually appear automatically with the projections of $\varnothing\rho'_{ab}$ and $\varnothing c_{s ab} \varnothing t'_{ab}$ by the curved $\varnothing\hat{\rho}$ and $\varnothing\hat{c}_s\varnothing\hat{t}$ in Fig. 5.

Figure 8 contains the curved absolute intrinsic metric space $\varnothing\hat{\rho}$ and straight line absolute intrinsic metric time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ in the context of absolute intrinsic Riemannian metric spacetime geometry, with respect to 3-observers in the relative proper physical Euclidean 3-space \mathbb{E}'^3 . The intrinsic metric coordinate interval projection relations derivable from Fig. 8, from which absolute intrinsic metric line element must be derived on $(\varnothing\hat{\rho}, \varnothing c_{s ab} \varnothing t'_{ab})$, with respect to 3-observers in \mathbb{E}'^3 solely in that figure are

$$\varnothing\hat{c}_s d\varnothing\hat{t} = \varnothing c_{s ab} d\varnothing t'_{ab} \cos(-\varnothing\hat{\psi}_{P0}) \text{ and } d\varnothing\rho'_{ab} = d\varnothing\hat{\rho} \cos \varnothing\hat{\psi}_P ,$$

or

$$\varnothing c_{s ab} d\varnothing t'_{ab} = \varnothing\hat{c}_s d\varnothing\hat{t} \sec \varnothing\hat{\psi} ; \quad (48a)$$

$$d\varnothing\rho'_{ab} = d\varnothing\hat{\rho} \cos \varnothing\hat{\psi} . \quad (48b)$$

Equation (48a) derived by 3-observers in \mathbb{E}'^3 in Fig. 8 replaces Eq. (19a) derived by 1-observers in the proper time dimension $c_s t'$ in Fig. 5. Equations (48a) and (48b) are absolute intrinsic time dilation and absolute intrinsic length contraction formulae with respect to 3-observers in the proper physical 3-space \mathbb{E}'^3 , encompassed by the absolute intrinsic Riemannian spacetime geometry of Fig. 8.

The projective absolute proper intrinsic metric coordinate intervals $d\varnothing\rho'_{ab}$ and $\varnothing c_{s ab} d\varnothing t'_{ab}$ in Fig. 8 have been put into consideration in relations (48a) and (48b), while the projective 'non-metric' intrinsic coordinate intervals $\delta\varnothing\hat{\rho}$ and $\varnothing c_{s ab} \delta\varnothing t'_{ab}$ have been disregarded. Indeed the absolute intrinsic metric line element on the curved absolute intrinsic metric spacetime $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$, which is valid with respect to 3-observers in the relative proper Euclidean 3-space \mathbb{E}'^3 solely in Fig. 5, must be synthesized from the intrinsic metric coordinate interval projection relations (48a) and (48b) derived from Fig. 8. However the appropriate structure (or signature) of that absolute intrinsic line element to adopt is yet unknown and cannot be determined from Eqs. (48a) and (48b).

In order to determine the structure (or signature) of the absolute intrinsic line element and, consequently, of the absolute intrinsic metric tensor, on the curved 'two-dimensional' absolute intrinsic metric spacetime $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$ in Fig. 5, which is valid with respect to 3-observers in \mathbb{E}'^3 solely, we must first determine which of absolute intrinsic local Euclidean invariance (A \varnothing LEI) and absolute intrinsic local Lorentz invariance (A \varnothing LLI) on $(\varnothing\hat{\rho}, \varnothing c_{s ab} \varnothing t'_{ab})$ is valid with respect to 3-observers in \mathbb{E}'^3 solely in Fig. 8. Thus let us take into account the projective 'non-metric' components in the intrinsic coordinate projection relations that can be derived from Fig. 8 to have

$$\varnothing\hat{c}_s d\varnothing\hat{t} = \varnothing c_{s ab} d\varnothing t'_{ab} \cos(-\varnothing\hat{\psi}_{P0}); \quad \delta\varnothing\hat{\rho} = d\varnothing\hat{\rho} \sin \varnothing\hat{\psi}_P ,$$

or

$$\varnothing c_{s ab} d\varnothing t'_{ab} = \varnothing\hat{c}_s d\varnothing\hat{t} \sec \varnothing\hat{\psi}_{P0} ; \quad \delta\varnothing\hat{\rho} = d\varnothing\hat{\rho} \sin \varnothing\hat{\psi}_P , \quad (49a)$$

along the vertical and

$$d\varnothing\rho'_{ab} = d\varnothing\hat{\rho} \cos \varnothing\hat{\psi}_P ; \quad \varnothing c_{s ab} \delta\varnothing t'_{ab} = \varnothing c_{s ab} d\varnothing t'_{ab} \sin(-\varnothing\hat{\psi}_{P0}) ,$$

or

$$d\varnothing\rho'_{ab} = d\varnothing\hat{\rho} \cos \varnothing\hat{\psi}_P ; \varnothing c_{sab} \delta\varnothing t'_{ab} = -\varnothing c_{sab} d\varnothing t'_{ab} \sin \varnothing\hat{\psi}_{P0} , \quad (49b)$$

or

$$d\varnothing\rho'_{ab} = d\varnothing\hat{\rho} \cos \varnothing\hat{\psi}_P ; \varnothing c_{sab} \delta\varnothing t'_{ab} = -\varnothing \hat{c}_s d\varnothing \hat{t} \tan \varnothing\hat{\psi}_{P0} , \quad (49c)$$

along the horizontal, where Eq. (49a) has been used in the second equation between systems (49b) and (49c).

The intrinsic coordinate projection relations of systems (49a) and (49c) derived from Fig. 8 are valid with respect to 3-observers in the relative proper physical Euclidean 3-space \mathbb{E}^3 solely in that figure. They correspond to systems (20a) and (20b) derived from Fig. 5, which are valid with respect to 1-observers in $c_s t'$ and 3-observers in \mathbb{E}^3 respectively.

Now the only absolute intrinsic space coordinate interval $d\varnothing\hat{\rho}$ about point P along the absolute intrinsic space $\varnothing\hat{\rho}$, when $\varnothing\hat{\rho}$ was a straight line along the horizontal in Fig. 7, becomes replaced by components $d\varnothing\rho'_{ab}$ and $\varnothing c \delta\varnothing t'_{ab}$ projected along the horizontal in Fig. 8, upon the evolution of the geometry of Fig. 8 from Fig. 7. There is invariance of the squares of intrinsic coordinate intervals along the horizontal between Fig. 7 and Fig. 8, expressed as follows,

$$(d\varnothing\rho'_{ab})^2 + \varnothing c_{sab}^2 (\delta\varnothing t'_{ab})^2 = d\varnothing\hat{\rho}^2 ,$$

or

$$\begin{aligned} (d\varnothing\rho'_{ab})^2 &= d\varnothing\hat{\rho}^2 - \varnothing c_{sab}^2 (\delta\varnothing t'_{ab})^2 \\ &= (d\varnothing\rho'_{ab})^2 \sec^2 \varnothing\hat{\psi}_P - \varnothing c_{sab}^2 (d\varnothing t'_{ab})^2 \sin^2 \varnothing\hat{\psi}_{P0} , \end{aligned} \quad (50)$$

where the second equation of system (49b) has been used. In effect, this equation expresses invariance of partial intrinsic line element between the curved 'one-dimensional' absolute intrinsic metric space $\varnothing\hat{\rho}$ and its projective straight line absolute proper intrinsic metric space $\varnothing\rho'_{ab}$ in Fig. 8.

The following relation likewise obtains between the absolute proper intrinsic coordinate interval $\varnothing c_{sab} d\varnothing t'_{ab}$ along the curved absolute proper intrinsic metric time dimension $\varnothing c_{sab} \varnothing t'_{ab}$ and the absolute intrinsic coordinate intervals, $\varnothing \hat{c}_s d\varnothing \hat{t}$ and $\delta\varnothing\hat{\rho}$, projected into the straight line absolute intrinsic metric time 'dimension' $\varnothing \hat{c}_s \varnothing \hat{t}$ along the vertical in the upper half of Fig. 8

$$\begin{aligned} \varnothing c_{sab}^2 (d\varnothing t'_{ab})^2 &= \varnothing \hat{c}_s^2 d\varnothing \hat{t}^2 + \delta\varnothing\hat{\rho}^2 ; \\ &= \varnothing c_{sab}^2 (d\varnothing t'_{ab})^2 \cos^2 \varnothing\hat{\psi}_{P0} + d\varnothing\hat{\rho}^2 \sin^2 \varnothing\hat{\psi}_P ; \\ &= \varnothing c_{sab}^2 (d\varnothing t'_{ab})^2 \cos^2 \varnothing\hat{\psi}_{P0} + (d\varnothing\rho'_{ab})^2 \tan^2 \varnothing\hat{\psi}_P , \end{aligned} \quad (51)$$

where the first equation of system (49b) has been used between the last two lines of equations.

Again Eq. (51) expresses invariance of partial intrinsic line element between the curved one-dimensional absolute proper intrinsic time dimension $\varnothing c_{sab} \varnothing t'_{ab}$ and the straight line absolute intrinsic time 'dimension' $\varnothing \hat{c}_s \varnothing \hat{t}$ along the vertical in Fig. 7. Both relations (50) and (51) have been derived by or with respect to 3-observers in the relative proper Euclidean 3-space \mathbb{E}^3 solely in Fig. 8, with respect to whom Fig. 8 is valid.

Now the addition of Eqs. (50) and (51) does not lead to absolute intrinsic local Euclidean invariance (A \varnothing LEI), as can be easily verified. It may be recalled that the addition of the corresponding Eqs. (21a) and (21b) derived from Fig. 5, leads to absolute intrinsic local Euclidean invariance expressed by Eq. (22).

On the other hand, let us subtract Eq. (50) from Eq. (51) to have as follows

$$\begin{aligned} \varnothing c_{s'ab}^2 (d\varnothing t'_{ab})^2 - (d\varnothing \rho'_{ab})^2 &= \varnothing c_{s'ab}^2 (d\varnothing t'_{ab})^2 \cos^2 \varnothing \hat{\psi} + (d\varnothing \rho'_{ab})^2 \tan^2 \varnothing \hat{\psi} \\ &\quad - (d\varnothing \rho'_{ab})^2 \sec^2 \varnothing \hat{\psi} - \varnothing c_{s'ab}^2 (d\varnothing t'_{ab})^2 \sin^2 \varnothing \hat{\psi}, \end{aligned} \quad (52)$$

where the fact that, $\varnothing \hat{\psi}_P = \varnothing \hat{\psi}_{P_0} \equiv \varnothing \hat{\psi}$, has been used. Equation (52) is given as follows by associating like terms at the right-hand side

$$\begin{aligned} \varnothing c_{s'ab}^2 (d\varnothing t'_{ab})^2 - (d\varnothing \rho'_{ab})^2 &= \varnothing c_{s'ab}^2 (d\varnothing t'_{ab})^2 (\cos^2 \varnothing \hat{\psi} + \sin^2 \varnothing \hat{\psi}) \\ &\quad - (d\varnothing \rho'_{ab})^2 (\sec^2 \varnothing \hat{\psi} - \tan^2 \varnothing \hat{\psi}). \end{aligned} \quad (53)$$

Equation (53) expresses absolute intrinsic local Lorentz invariance (A \varnothing LLI) in terms of absolute proper intrinsic metric coordinate intervals by virtue of the expressions, $\cos^2 \varnothing \hat{\psi} + \sin^2 \varnothing \hat{\psi} = 1$ and $\sec^2 \varnothing \hat{\psi} - \tan^2 \varnothing \hat{\psi} = 1$.

Now the invariance of intrinsic line element between the absolute intrinsic spacetime $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$ and the absolute proper intrinsic spacetime $(\varnothing \rho'_{ab}, \varnothing c_{s'ab} \varnothing t'_{ab})$ in Fig. 8 allows us to write the following

$$d\varnothing \hat{s}^2 = (d\varnothing s'_{ab})^2$$

or

$$\varnothing \hat{c}_s^2 d\varnothing \hat{t}^2 - d\varnothing \hat{\rho}^2 = \varnothing c_{s'ab}^2 (d\varnothing t'_{ab})^2 - (d\varnothing \rho'_{ab})^2, \quad (54)$$

where $d\varnothing \hat{s}$ pertains to the 'two-dimensional' absolute intrinsic metric spacetime $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$ bounded by curved $\varnothing \hat{\rho}$ and straight line $\varnothing \hat{c}_s \varnothing \hat{t}$, while $d\varnothing s'_{ab}$ pertains to $(\varnothing \rho'_{ab}, \varnothing c_{s'ab} \varnothing t'_{ab})$ in Fig. 8. Equation (54) is still a statement of absolute intrinsic local Lorentz invariance (A \varnothing LLI) on the curved $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$ with respect to 3-observers in \mathbb{E}^3 solely in Fig. 5, where Fig. 8 must be used to derive it.

It follows from Eq. (54) that the absolute proper intrinsic coordinate intervals, $d\varnothing \rho'_{ab}$ and $\varnothing c_{s'ab} d\varnothing t'_{ab}$, can be replaced with absolute intrinsic coordinate intervals, $d\varnothing \hat{\rho}$ and $\varnothing \hat{c}_s d\varnothing \hat{t}$, respectively in Eq. (52) to have

$$\begin{aligned} \varnothing \hat{c}_s^2 d\varnothing \hat{t}^2 - d\varnothing \hat{\rho}^2 &= \varnothing \hat{c}_s^2 d\varnothing \hat{t}^2 (\cos^2 \varnothing \hat{\psi} + \sin^2 \varnothing \hat{\psi}) \\ &\quad - d\varnothing \hat{\rho}^2 (\sec^2 \varnothing \hat{\psi} - \tan^2 \varnothing \hat{\psi}). \end{aligned} \quad (55)$$

Again Eq. (55) expresses absolute intrinsic local Lorentz invariance (A \varnothing LLI) on $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$ with respect to 3-observer in \mathbb{E}^3 solely in Fig. 8, in terms of absolute intrinsic coordinate intervals of $(\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t})$, which is the appropriate thing to do. Let us replace $\varnothing \hat{c}_s^2 d\varnothing \hat{t}^2 - d\varnothing \hat{\rho}^2$ by $d\varnothing \hat{s}^2$ at the left-hand side of (55) to have

$$\begin{aligned} d\varnothing \hat{s}^2 &= \varnothing \hat{c}_s^2 d\varnothing \hat{t}^2 (\cos^2 \varnothing \hat{\psi} + \sin^2 \varnothing \hat{\psi}) \\ &\quad - d\varnothing \hat{\rho}^2 (\sec^2 \varnothing \hat{\psi} - \tan^2 \varnothing \hat{\psi}) \end{aligned} \quad (56)$$

or

$$d\varnothing \hat{s}^2 = \varnothing \hat{c}_s^2 d\varnothing \hat{t}^2 - d\varnothing \hat{\rho}^2. \quad (57)$$

The absolute intrinsic Lorentzian line element (56) or (57) obtains at every point along the curved $\varnothing \hat{\rho}$ and the symmetry-partner point along the straight line absolute intrinsic time 'dimension' $\varnothing \hat{c}_s \varnothing \hat{t}$, with respect to 3-observers in \mathbb{E}^3 in Fig. 8, in so far as both the metric and 'non-metric' intrinsic coordinate interval projections are taken into account in deriving intrinsic coordinate projection relations from Fig. 8, as done in systems (49a-c) and in Eqs. (50) and (51).

In brief, it is absolute intrinsic local Lorentz invariance (A \varnothing LLI) (and not absolute intrinsic local Euclidean invariance (A \varnothing LEI)) that obtains on the 'two-dimensional' absolute intrinsic spacetime bounded by curved $\varnothing \hat{\rho}$ and straight line $\varnothing \hat{c}_s \varnothing \hat{t}$, with respect to 3-observers in in the proper metric

Euclidean 3-space \mathbb{E}^3 in Fig. 8, in so far as both the metric and the ‘non-metric’ intrinsic coordinate interval projections are taken into account in deriving intrinsic coordinate projection relations from Fig. 8. It can also be said that $A\emptyset LLI$ obtains on the curved ‘two-dimensional’ absolute intrinsic metric spacetime $(\emptyset\hat{\rho}, \emptyset\hat{c}_s\emptyset\hat{t})$ with respect to 3-observers in \mathbb{E}^3 solely in Fig. 5, since Fig. 8 is Fig. 5 made valid with respect to 3-observers in \mathbb{E}^3 solely.

Apart from deriving absolute intrinsic local Lorentz invariance ($A\emptyset LLI$) on the curved ‘two-dimensional’ absolute intrinsic metric spacetime $(\emptyset\hat{\rho}, \emptyset\hat{c}_s\emptyset\hat{t})$ with respect to 3-observers in \mathbb{E}^3 solely by making use of both the metric and non-metric intrinsic coordinate projections, as done above, the $A\emptyset LLI$ on $(\emptyset\hat{\rho}, \emptyset\hat{c}_s\emptyset\hat{t})$ with respect to these observers is required in order to be able to re-write the absolute intrinsic line element (54) in the appropriate form of Eq. (55).

We have thus shown that the natural absolute intrinsic local Euclidean invariance ($A\emptyset LEI$) on the curved ‘two-dimensional’ absolute intrinsic spacetime $(\emptyset\hat{\rho}, \emptyset\hat{c}_s\emptyset\hat{t})$, which is valid partially with respect to 3-observers in the proper Euclidean 3-space \mathbb{E}^3 and partially with respect to 1-observers in the proper time dimension $c_s t'$ in Fig. 5, becomes required absolute intrinsic local Lorentz invariance ($A\emptyset LLI$) on the curved $(\emptyset\hat{\rho}, \emptyset\hat{c}_s\emptyset\hat{t})$, with respect to 3-observers in \mathbb{E}^3 solely, with respect to whom the diagram of Fig. 8 must be drawn. It shall be shown with further development elsewhere that $A\emptyset LLI$ is trivially satisfied, apart from the consideration of both the projective metric and non-metric intrinsic coordinate intervals as done above, because of the natural absolutism of intrinsic coordinates namely, $\emptyset x'^k_{ab} = \emptyset \hat{x}^k$; $k = 0, 1$, in the absolute intrinsic Riemannian metric spacetime geometry.

Now let us separate $d\emptyset\hat{s}^2$ in Eq. (56) into the metric and ‘non-metric’ components as

$$\begin{aligned} d\emptyset\hat{s}^2 &= d\emptyset\hat{s}_m^2 + d\emptyset\hat{s}_{nm}^2 \\ &= \sum_{i,j=0}^1 \emptyset\hat{g}_{ij} d\emptyset\hat{x}^i d\emptyset\hat{x}^j - \sum_{i,j=0}^1 \emptyset\hat{R}_{ij} d\emptyset\hat{x}^i d\emptyset\hat{x}^j \end{aligned} \quad (58)$$

or

$$\begin{aligned} d\emptyset\hat{s}^2 &= \left(\emptyset\hat{c}_s^2 (d\emptyset\hat{t})^2 \cos^2 \emptyset\hat{\psi} - (d\emptyset\hat{\rho})^2 \sec^2 \emptyset\hat{\psi} \right) \\ &\quad - \left(-\emptyset\hat{c}_s^2 (d\emptyset\hat{t})^2 \sin^2 \emptyset\hat{\psi} - (d\emptyset\hat{\rho})^2 \tan^2 \emptyset\hat{\psi} \right). \end{aligned} \quad (59)$$

The absolute intrinsic line element without star label on the curved ‘two-dimensional’ absolute intrinsic metric spacetime $(\emptyset\hat{\rho}, \emptyset c_s \emptyset\hat{t})$ that is valid with respect to 3-observers in \mathbb{E}^3 solely in Fig. 5 (which is the same as the absolute intrinsic line element on $(\emptyset\hat{\rho}, \emptyset\hat{c}_{s_{ab}}\emptyset t'_{ab})$ in Fig. 8), which follows from Eqs. (58) and (59) is the following

$$\begin{aligned} d\emptyset\hat{s}_m^2 &= \sum_{i,j=0}^1 \emptyset\hat{g}_{ij} d\emptyset\hat{x}^i d\emptyset\hat{x}^j \\ &= \emptyset\hat{g}_{00} \emptyset\hat{c}_s^2 d\emptyset\hat{t}^2 + \emptyset\hat{g}_{11} d\emptyset\hat{\rho}^2 \end{aligned} \quad (60)$$

$$= \cos^2 \emptyset\hat{\psi} \emptyset\hat{c}_s^2 d\emptyset\hat{t}^2 - \sec^2 \emptyset\hat{\psi} d\emptyset\hat{\rho}^2 \quad (61)$$

$$= (1 - \emptyset\hat{k}^2) \emptyset\hat{c}_s^2 d\emptyset\hat{t}^2 - \frac{d\emptyset\hat{\rho}^2}{1 - \emptyset\hat{k}^2} \quad (62)$$

The derived hyperbolic absolute intrinsic line element of Eq. (61) or (62) on the curved ‘two-dimensional’ absolute intrinsic metric spacetime $(\emptyset\hat{\rho}, \emptyset c_s \emptyset\hat{t})$ in Fig. 8, which is valid with respect to 3-observers in the relative proper metric Euclidean 3-space \mathbb{E}^3 solely (or on the curved $(\emptyset\hat{\rho}, \emptyset\hat{c}_s\emptyset\hat{t})$ with respect

to 3-observers in \mathbb{E}'^3 in Fig. 5), implies the following hyperbolic absolute intrinsic metric tensor with respect to 3-observers in \mathbb{E}'^3

$$\varnothing \hat{g}_{ij} = \begin{pmatrix} \cos^2 \varnothing \hat{\psi} & 0 \\ 0 & -\sec^2 \varnothing \hat{\psi} \end{pmatrix}, \quad (63)$$

or

$$\varnothing \hat{g}_{ij} = \begin{pmatrix} 1 - \varnothing \hat{k}^2 & 0 \\ 0 & -\frac{1}{1 - \varnothing \hat{k}^2} \end{pmatrix}. \quad (64)$$

The absolute intrinsic line element (61) or (62) and the absolute intrinsic metric tensor (63) or (64), on the curved absolute intrinsic space - straight line absolute intrinsic time 'dimension' ($\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t}$) in Fig. 8, which are valid with respect to 3-observers in the proper physical Euclidean 3-space \mathbb{E}'^3 solely in that figure, are now hyperbolic as known for spacetime metrics. It can also be said that Eqs. (61) – (64) are valid on the curved 'two-dimensional' absolute intrinsic metric spacetime ($\varnothing \hat{\rho}, \varnothing \hat{c}_s \varnothing \hat{t}$) in Fig. 5, with respect to 3-observers in the proper physical Euclidean 3-space \mathbb{E}'^3 solely in that figure.

Equation (60) or (61) and Eq. (63) or (64) give the final forms of the absolute intrinsic line element and absolute intrinsic metric tensor in the context of 'two-dimensional' absolute intrinsic Riemannian spacetime geometry (or absolute Riemannian nospace-notime geometry) in our universe. The absolute intrinsic curvature parameter $\varnothing \hat{k}$ that appears in them shall be related to the absolute intrinsic parameters of the metric force field that gives rise to absolute intrinsic Riemannian spacetime geometry within a region of the universal spacetime elsewhere.

The absolute intrinsic metric tensor (without star label) $\varnothing \hat{g}_{ij}$ of Eq. (63) or (64) (on a manifold of the type \mathbb{M}^{p+q} , which is $\varnothing \hat{\mathbb{M}}^{1+1}$ in the present case), is the modified form of the starred absolute intrinsic metric tensor $\varnothing \hat{g}_{ij}^*$ of Eq. (32) or (33) (on a manifold of the type \mathbb{M}^p , which is $\varnothing \hat{\mathbb{M}}^2$ in the present case). The components of $\varnothing \hat{g}_{ij}^*$ and $\varnothing \hat{g}_{ij}$ are related by comparing Eqs. (33) and (64) as follows

$$\varnothing \hat{g}_{00} = \frac{1}{\varnothing \hat{g}_{00}^*}; \varnothing \hat{g}_{11} = -\varnothing \hat{g}_{11}^*; \varnothing \hat{g}_{ij} = \varnothing \hat{g}_{ij}^* = 0; i \neq j. \quad (65a)$$

The following relations also follow among the components of $\varnothing \hat{g}_{ij}^*$ in Eq. (33) and among the components of $\varnothing \hat{g}_{ij}$ in Eq. (63)

$$\varnothing \hat{g}_{11}^* = \varnothing \hat{g}_{00}^*; \varnothing \hat{g}_{11} = -\frac{1}{\varnothing \hat{g}_{00}}. \quad (65b)$$

The validity of systems (65a) and (65b) in all situations is guaranteed by the fact that there is a perfect symmetry of state between the positive time-universe and our universe and, indeed, among the four universes isolated in [3–6], as mentioned earlier. This fact guarantees that the curvature of the absolute intrinsic space $\varnothing \hat{\rho}$ relative to the absolute proper intrinsic space $\varnothing \hat{\rho}'_{ab}$ at every point along $\varnothing \hat{\rho}$ is identical to the curvature of the absolute intrinsic time 'dimension' $\varnothing \hat{c}_s \varnothing \hat{t}$ relative to the absolute proper intrinsic time dimension $\varnothing \hat{c}_{s'ab} \varnothing \hat{t}'_{ab}$ at the symmetry-partner point along $\varnothing \hat{c}_s \varnothing \hat{t}$ in Fig. 5. Hence the absolute intrinsic curvature parameter $\varnothing \hat{k}_P$ at point P on curved $\varnothing \hat{\rho}$ is identical to the absolute intrinsic curvature parameter $\varnothing \hat{k}_{P^0}$ at the symmetry-partner point P^0 of the curved $\varnothing \hat{c}_s \varnothing \hat{t}$. That is, $\varnothing \hat{\psi}_P = \varnothing \hat{\psi}_{P^0} \equiv \varnothing \hat{\psi}$, hence, $\varnothing \hat{k}_P = \varnothing \hat{k}_{P^0} \equiv \varnothing \hat{k}$, in Fig. 5, as mentioned earlier, and this is true in all situations and implies that systems (65a) and (65b) are true in all situations.

In obtaining the final absolute intrinsic metric tensor $\varnothing \hat{g}_{ij}$ without star label of Eq. (63) or (64) tensorially, one must solve the pair of starred absolute intrinsic tensor equations (34) and (38) simultaneously to obtain the starred absolute intrinsic metric tensor $\varnothing \hat{g}_{ij}^*$ of Eq. (33) and the starred absolute intrinsic Ricci tensor $\varnothing \hat{R}_{ij}^*$ of Eq. (39).

One must then apply relations (65a) and (65b) to obtain the absolute intrinsic metric tensor without star label $\varnothing\hat{g}_{ij}$ from the starred absolute intrinsic metric tensor $\varnothing\hat{g}_{ij}^*$ so obtained.

In order to obtain the absolute intrinsic Ricci tensor without star label $\varnothing\hat{R}_{ij}$, which is compatible with the absolute intrinsic metric tensor without star label $\varnothing\hat{g}_{ij}$, obtained from the program in the preceding paragraph, we shall make use of the validity of absolute intrinsic local Lorentz invariance (A \varnothing LLI) on $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$, with respect to 3-observers in the proper physical Euclidean 3-space \mathbb{E}^3 in Fig. 8, demonstrated above. This implies that Eq. (34) must now be written in terms of absolute intrinsic tensors without star label $\varnothing\hat{g}_{ij}$ and $\varnothing\hat{R}_{ij}$ and with the Euclidean metric tensor δ_{ij} in that equation replaced with the Lorentzian metric tensor η_{ij} . In other words, the following absolute intrinsic tensorial statement of absolute intrinsic local Lorentz invariance on the curved $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$ with respect to 3-observers in \mathbb{E}^3 solely must be satisfied,

$$\varnothing\hat{g}_{ij} - \varnothing\hat{R}_{ij} = \eta_{ij} \text{ (A}\varnothing\text{LLI)} . \tag{66}$$

With $\varnothing\hat{g}_{ij}$ given by Eq. (63) or (64), the absolute intrinsic Ricci tensor without star label $\varnothing\hat{R}_{ij}$ that satisfies Eq. (66) is the following

$$\varnothing\hat{R}_{ij} = \begin{pmatrix} -\sin^2 \varnothing\hat{\psi} & 0 \\ 0 & -\tan^2 \varnothing\hat{\psi} \end{pmatrix} ; \tag{67}$$

$$= \begin{pmatrix} -\varnothing\hat{k}^2 & 0 \\ 0 & -\frac{\varnothing\hat{k}^2}{1 - \varnothing\hat{k}^2} \end{pmatrix} . \tag{68}$$

Equation (67) and, hence, Eq. (68), can also be written from the second terms at the right-hand sides of Eqs. (58) and (59).

Now let us consider a situation where a pair of ‘two-dimensional’ absolute intrinsic metric spacetimes coexist. One will naturally be curved relative to the other as illustrated in Fig. 6. The lower half of Fig. 6 is valid with respect to 3-observers in \mathbb{E}^3 , while the upper half is valid with respect to 1-observers in $c_s t'$. In order to make Fig. 6 valid with respect to 3-observers in \mathbb{E}^3 solely, it must be modified as Fig. 9, which follows from the explanation for drawing Fig. 8 from Fig. 5.

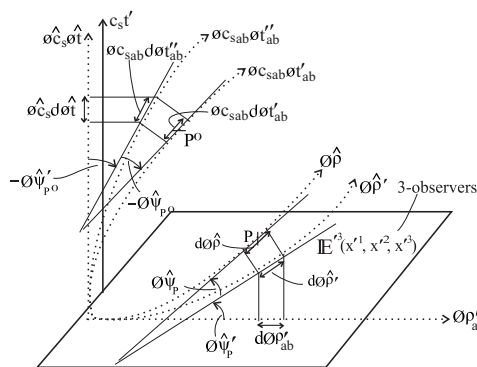


Fig. 9. Deriving resultant absolute intrinsic coordinate projections with respect to 3-observers in the underlying relative proper metric Euclidean 3-space solely, when two curved absolute intrinsic metric spacetimes (or absolute intrinsic Riemannian metric spacetimes) co-exist

The resultant absolute intrinsic metric coordinate interval projection relations, or the resultant absolute intrinsic length contraction and resultant absolute intrinsic time dilation formulae, which are valid with respect to 3-observers in \mathbb{E}^{13} solely in Fig. 9 are the following

$$\begin{aligned} d\varnothing\rho'_{ab} &= d\varnothing\hat{\rho} \cos \varnothing\hat{\psi}_{\text{res}} \\ &= d\varnothing\hat{\rho} \cos \varnothing\hat{\psi}' \cos \varnothing\hat{\psi} \\ &= d\varnothing\hat{\rho}(1 - \varnothing\hat{k}'^2)^{1/2}(1 - \varnothing\hat{k}^2)^{1/2} \end{aligned} \quad (69)$$

and

$$\begin{aligned} \varnothing c_{sab}d\varnothing t'_{ab} &= \varnothing\hat{c}_s d\varnothing\hat{t} \sec \varnothing\hat{\psi}_{\text{res}} \\ &= \varnothing\hat{c}_s d\varnothing\hat{t} \sec \varnothing\hat{\psi}' \sec \varnothing\hat{\psi} \\ &= \varnothing\hat{c}_s d\varnothing\hat{t}(1 - \varnothing\hat{k}'^2)^{-1/2} \times \\ &\quad (1 - \varnothing\hat{k}^2)^{-1/2}. \end{aligned} \quad (70)$$

The resultant absolute intrinsic metric tensor without star label $\varnothing\hat{g}_{ij}$ and the resultant absolute intrinsic Ricci tensor without star label $\varnothing\hat{R}_{ij}$, which are valid with respect to 3-observers in \mathbb{E}^{13} solely in Fig. 8, are given by writing Eqs.(63) and (64) in terms of the resultant absolute intrinsic angle $\varnothing\hat{\psi}$ and resultant absolute intrinsic curvature parameter $\varnothing\hat{k}$ as follows

$$\varnothing\hat{g}_{ij} = \begin{pmatrix} 1 - \sin^2 \varnothing\hat{\psi} & 0 \\ 0 & -\frac{1}{1 - \sin^2 \varnothing\hat{\psi}} \end{pmatrix} \quad (71)$$

where, $\sin^2 \varnothing\hat{\psi} = \sin^2 \varnothing\hat{\psi}' + \sin^2 \varnothing\hat{\psi}$, as follows from the derived relation (90) of [2]. Equation (71) corresponds to the following in terms of resultant absolute intrinsic curvature parameter,

$$\varnothing\hat{g}_{ij} = \begin{pmatrix} 1 - (\varnothing\hat{k})^2 & 0 \\ 0 & -\frac{1}{1 - (\varnothing\hat{k})^2} \end{pmatrix}; \quad (72)$$

$$= \begin{pmatrix} 1 - (\varnothing\hat{k}')^2 - \varnothing\hat{k}^2 & 0 \\ 0 & -\frac{1}{1 - (\varnothing\hat{k}')^2 - \varnothing\hat{k}^2} \end{pmatrix}, \quad (73)$$

where, $(\varnothing\hat{k})^2 = (\varnothing\hat{k}')^2 + \varnothing\hat{k}^2$, as derived in [2] and presented as Eq. (91) of that paper.

And by writing equations (67) and (68) in terms of the resultant absolute intrinsic angle $\varnothing\hat{\psi}$ and resultant absolute intrinsic curvature parameter $\varnothing\hat{k}$ we have

$$\varnothing\hat{R}_{ij} = \begin{pmatrix} -\sin^2 \varnothing\hat{\psi} & 0 \\ 0 & -\frac{\sin^2 \varnothing\hat{\psi}}{1 - \sin^2 \varnothing\hat{\psi}} \end{pmatrix} \quad (74)$$

or

$$\varnothing\hat{R}_{ij} = \begin{pmatrix} -(\varnothing\hat{k})^2 & 0 \\ 0 & -\frac{(\varnothing\hat{k})^2}{1 - (\varnothing\hat{k})^2} \end{pmatrix} \quad (75)$$

$$= \begin{pmatrix} -(\varnothing\hat{k}')^2 - \varnothing\hat{k}^2 & 0 \\ 0 & -\frac{(\varnothing\hat{k}')^2 + \varnothing\hat{k}^2}{1 - (\varnothing\hat{k}')^2 - \varnothing\hat{k}^2} \end{pmatrix}. \quad (76)$$

Thus the resultant absolute intrinsic line element on the upper curved ‘two-dimensional’ absolute intrinsic metric spacetime $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$ in Fig. 6, which is valid with respect to 3-observers in \mathbb{E}^3 solely in that figure, derived via Fig. 9 is the following

$$\begin{aligned} d\varnothing\hat{s}^2 &= \varnothing\hat{g}_{00}\varnothing\hat{c}_s^2d\varnothing\hat{t}^2 - \varnothing\hat{g}_{11}d\varnothing\hat{\rho}^2 \\ &= (1 - \sin^2\varnothing\hat{\psi}' - \sin^2\varnothing\hat{\psi})\varnothing\hat{c}_s^2d\varnothing\hat{t}^2 \\ &\quad - \frac{d\varnothing\hat{\rho}^2}{1 - \sin^2\varnothing\hat{\psi}' - \sin^2\varnothing\hat{\psi}}, \end{aligned} \quad (77)$$

or

$$d\varnothing\hat{s}^2 = \frac{(1 - (\varnothing\hat{k}')^2 - \varnothing\hat{k}^2)\varnothing\hat{c}_s^2d\varnothing\hat{t}^2}{1 - (\varnothing\hat{k}')^2 - \varnothing\hat{k}^2}. \quad (78)$$

The extension of relations (69) through (78) to the situation where three and a larger number of curved ‘two-dimensional’ absolute intrinsic metric spacetimes (or absolute intrinsic Riemannian metric spacetimes) co-exist is straight forward.

3 ISOLATING NON UNIFORM ABSOLUTE INTRINSIC ‘STATIC FLOW’ SPEED ALONG THE CURVED ABSOLUTE INTRINSIC METRIC SPACE AND CURVED ABSOLUTE INTRINSIC METRIC TIME ‘DIMENSION’ IN A LONG-RANGE METRIC FORCE FIELD

static absolute intrinsic metric time ‘dimension’ $\varnothing\hat{c}_s\varnothing\hat{t}$ at point P^0 along $\varnothing\hat{c}_s\varnothing\hat{t}$, spans interval $\varnothing c_{sab}d\varnothing t'_{ab}$ of $\varnothing c_{sab}\varnothing t'_{ab}$ along the vertical and interval $d\varnothing\hat{\rho}$ of $\varnothing\hat{\rho}$ along the horizontal in Fig. 10a. The curved ‘dimension’ $\varnothing\hat{c}_s\varnothing\hat{t}$ possesses absolute intrinsic static geodetic flow speed $\varnothing\hat{c}_s$ at every point along its ‘length, which is not made manifested in its absolute intrinsic geodetic flow. Hence $\varnothing\hat{c}_s\varnothing\hat{t}$ is a static absolute intrinsic metric time ‘dimension’.

The trigonometric sine ratio of the absolute intrinsic angle $\varnothing\hat{\psi}_{P^0}$ of inclination of the curved $\varnothing\hat{c}_s\varnothing\hat{t}$ to $\varnothing c_{sab}\varnothing t'_{ab}$ along the vertical at point P^0 along $\varnothing\hat{c}_s\varnothing\hat{t}$ is given as

$$\sin\varnothing\hat{\psi}_{P^0} = \frac{d\varnothing\hat{\rho}}{\varnothing\hat{c}_sd\varnothing\hat{t}} = \frac{\varnothing\hat{V}_{m,P^0}}{\varnothing\hat{c}_s}, \quad (79)$$

Figs. 10a and 10b are valid with respect to 1-observers in the static relative proper metric time dimension $c_s t'$ of our universe and 1-observers⁰ in the static relative proper metric time dimension $c_s t'^{0'}$ of the positive time-universe respectively, as indicated. The $c_s t'$ and $c_s t'^{0'}$ are referred to as static because, although they possess constant static geodetic flow speed c_s at every point along their lengths, they are not in geodetic flow, as explained in sub-section 1.4 of [3].

where, $d\varnothing\hat{\rho}/d\varnothing\hat{t} = \varnothing\hat{V}_{m,P^0}$, shall be referred to as absolute intrinsic ‘static flow’ speed. The metric force field establishes non-uniform ‘static flow’ speed $\varnothing\hat{V}_m$ along the curved absolute intrinsic metric time ‘dimension’ $\varnothing\hat{c}_s\varnothing\hat{t}$ with the value $\varnothing\hat{V}_{m,P^0}$ at point P^0 along $\varnothing\hat{c}_s\varnothing\hat{t}$, with respect to all 1-observers in the static relative proper metric time dimension $c_s t'$ of our universe along the vertical in Fig. 10a.

The elementary interval $\varnothing\hat{c}_sd\varnothing\hat{t}$ of the curved

The trigonometric sine ratio of the absolute intrinsic angle $\varnothing\hat{\psi}_P$ of inclination of the static

curved $\varnothing\hat{c}_s\varnothing\hat{t}^0$ relative to $\varnothing c_{sab}\varnothing t^{0'}_{ab}$ along the horizontal at point P along $\varnothing\hat{c}_s\varnothing\hat{t}^0$ in Fig. 10b, is likewise given as

$$\sin \varnothing\hat{\psi}_P = \frac{d\varnothing\hat{\rho}^0}{\varnothing\hat{c}_s d\varnothing\hat{t}^0} = \frac{\varnothing\hat{V}_{m,P}}{\varnothing\hat{c}_s}, \quad (80)$$

where, again, $d\varnothing\hat{\rho}^0/d\varnothing\hat{t}^0 = \varnothing\hat{V}_{m,P}$, is the absolute intrinsic 'static flow' speed of the curved absolute intrinsic static metric time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}^0$ at point P along $\varnothing\hat{c}_s\varnothing\hat{t}^0$, with respect to 1-observers⁰ in the relative proper static metric time dimension $c_s t^{0'}$ of the positive time-universe along the horizontal in Fig. 10b.

The absolute intrinsic 'static flow' speeds, $\varnothing\hat{V}_{m,P}$ and $\varnothing\hat{V}_{m,P^0}$, are established by the sources of the metric force fields located at the origins S and S^0 of the curved $\varnothing\hat{c}_s\varnothing\hat{t}$ and $\varnothing\hat{c}_s\varnothing\hat{t}^0$. Further difference between the non-uniform $\varnothing\hat{V}_m$, with maximum value $\varnothing\hat{c}_m$ and the absolute intrinsic static geodesic flow speed $\varnothing\hat{V}_0$ (or $\varnothing\hat{V}_s$) with maximum value $\varnothing\hat{c}_s$ (not established by the sources of the metric force field), where $\varnothing\hat{c}_m$ and $\varnothing\hat{c}_s$ have the same magnitude of $3 \times 10^3 \text{ m s}^{-1}$, shall be discussed shortly below.

Although the point P^0 of the curved static absolute intrinsic metric time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ possesses absolute intrinsic 'static flow speed' $\varnothing\hat{V}_{m,P^0}$ relative to 1-observers in $c_s t'$ in Fig.10a, and the point P of the curved static absolute

intrinsic time dimension $\varnothing\hat{c}_s\varnothing\hat{t}^0$ possesses absolute intrinsic 'static flow speed' $\varnothing\hat{V}_{m,P}$ relative to 1-observers in $c_s t^{0'}$ in Fig. 10b, the points P^0 of $\varnothing\hat{c}_s\varnothing\hat{t}$ and P of $\varnothing\hat{c}_s\varnothing\hat{t}^0$ are not in absolute intrinsic flow. This is so because $\varnothing\hat{V}_{m,P^0}$ and $\varnothing\hat{V}_{m,P}$ are static 'speeds' not made manifested in flow. This is in order, since $\varnothing\hat{c}_s\varnothing\hat{t}$ and $\varnothing\hat{c}_s\varnothing\hat{t}^0$ are static absolute intrinsic 'dimensions'.

Although $\varnothing\hat{V}_{m,P^0}$ and $\varnothing\hat{V}_{m,P}$ are intrinsic 'static flow' speeds, they are different from the absolute intrinsic static geodesic flow speed, $\varnothing\hat{V}_0$ (or $\varnothing\hat{V}_s$) with constant maximum value $\varnothing\hat{c}_s$, as discussed above. The divisor $\varnothing\hat{c}_s$ in $\varnothing\hat{V}_{m,P^0}/\varnothing\hat{c}_s$ and $\varnothing\hat{V}_{m,P}/\varnothing\hat{c}_s$ in Eqs. (79) and (80), should be the maximum $\varnothing\hat{c}_m$ over all possible values of the absolute intrinsic 'static flow' speed $\varnothing\hat{V}_{m,P^0}$ or $\varnothing\hat{V}_{m,P}$.

The maximum absolute intrinsic 'static flow' speed $\varnothing\hat{c}_m$ that must replace $\varnothing\hat{c}_s$ in Eqs. (79) and (80) is not the maximum absolute intrinsic dynamical $\varnothing\hat{c}_d$ or the maximum absolute intrinsic light speed $\varnothing\hat{c}_\gamma$ or the maximum absolute intrinsic static geodesic flow speed $\varnothing\hat{c}_s$. It is nevertheless a static (or non-dynamical) absolute intrinsic speed, with a magnitude of $3 \times 10^8 \text{ m s}^{-1}$ (not made manifested in absolute intrinsic flow of $\varnothing\hat{c}_s\varnothing\hat{t}$ and $\varnothing\hat{c}_s\varnothing\hat{t}^0$), like its reduced values, $\varnothing\hat{V}_{m,P^0}$ and $\varnothing\hat{V}_{m,P}$. The appropriate names and notations for $\varnothing\hat{V}_{m,P^0}$, $\varnothing\hat{V}_{m,P}$ and $\varnothing\hat{c}_m$, in the gravitational field, as a particular metric force field), shall be introduced elsewhere.

Equations (79) and (80) shall be corrected as follows by virtue of the foregoing discussions

$$\sin \varnothing\hat{\psi}_{m,P^0} = \frac{d\varnothing\hat{\rho}}{\varnothing\hat{c}_s d\varnothing\hat{t}} = \frac{\varnothing\hat{V}_{m,P^0}}{\varnothing\hat{c}_m} \quad (81)$$

and

$$\sin \varnothing\hat{\psi}_P = \frac{d\varnothing\hat{\rho}^0}{\varnothing\hat{c}_s d\varnothing\hat{t}^0} = \frac{\varnothing\hat{V}_{m,P}}{\varnothing\hat{c}_m}. \quad (82)$$

The full explanation of the appropriate equations (81) and (82) shall be given elsewhere. There will be found a new curved (active) static absolute intrinsic metric spacetime ($\varnothing\hat{\rho}_a, \varnothing\hat{c}_m\varnothing\hat{t}_a$), which is embedded in the curved (passive) static absolute intrinsic metric spacetimes ($\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t}$) in Fig. 10a and its counterpart Fig. 10b. Equation (79) will actually be, $d\varnothing\hat{\rho}_a/\varnothing\hat{c}_m d\varnothing\hat{t}_a = \varnothing\hat{V}_{m,P}/\varnothing\hat{c}_m$ in the more complete diagram.

The pair of points P^0 along the curved $\varnothing\hat{c}_s\varnothing\hat{t}$ in Fig. 10a and point P along the curved $\varnothing\hat{c}_s\varnothing\hat{t}^0$ in Fig. 10b are symmetry-partner points. Another pair of symmetry-partner points Q^0 along the curved

$\partial \hat{c}_s \partial \hat{t}$ in Fig. 10a and Q along the curved $\partial \hat{c}_s \partial \hat{t}^0$ in Fig. 10b, likewise possesses absolute intrinsic speeds $\partial \hat{V}_{m,Q^0}$ relative to 1-observers in $c_s t'$ in Fig. 10a and $\partial \hat{V}_{m,Q}$ relative to 1-observers in $c_s t^{0'}$ in Fig. 10b respectively. The absolute intrinsic speeds, $\partial \hat{V}_{m,P^0}$ and $\partial \hat{V}_{m,Q^0}$, along the curved $\partial \hat{c}_s \partial \hat{t}$ are illustrated in Fig. 11a, and the corresponding absolute intrinsic speeds, $\partial \hat{V}_{m,P}$ and $\partial \hat{V}_{m,Q}$, along the curved $\partial \hat{c}_s \partial \hat{t}^0$ are illustrated in Fig. 11b.

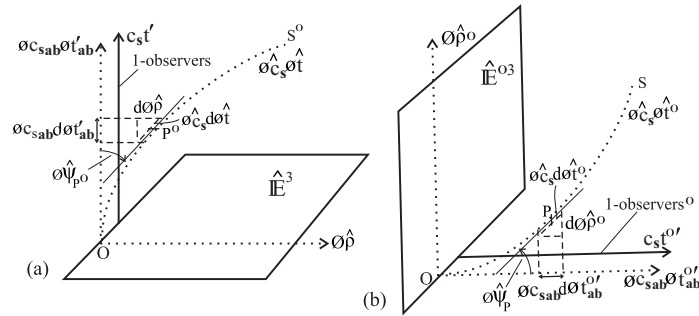


Fig. 10. Deriving non-uniform absolute intrinsic 'static flow' speeds established by the metric force field along the curved absolute intrinsic metric time 'dimensions' with respect to 1-observers in the relative proper static metric time dimensions of our universe and the positive time-universe

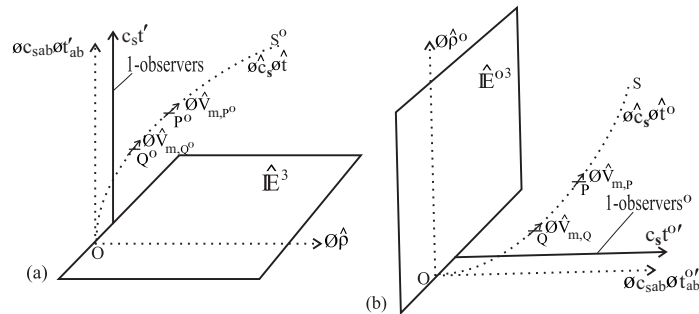


Fig. 11. Non-uniform absolute intrinsic 'static flow' speeds along curved absolute intrinsic metric time dimensions with respect to 1-observers in the relative proper metric time dimensions of our universe and positive time-universe, established by the sources of symmetry-partner long-range absolute intrinsic metric force fields located at symmetry-partner positions S and S^0 on the curved absolute intrinsic time dimensions.

The half-geometry of Fig. 10a with respect to 1-observers in the relative proper time dimension $c_s t'$ of our universe and the half-geometry of Fig. 11b with respect to 1-observers in the relative proper time dimension $c_s t^{0'}$ of the positive time-universe, co-exist and must be united into a singular diagram. In doing this and making the resulting diagram to contain the spacetime and intrinsic spacetime dimensions of our (or positive) universe solely, we must, as done in [3] let, $c_s t^0 \rightarrow \hat{E}^3$; $\partial c_{sab} \partial t_{ab}^{0'} \rightarrow \partial \hat{\rho}'_{ab}$ and $\partial \hat{c}_s \partial \hat{t}^0 \rightarrow \partial \hat{\rho}$, in Fig. 11b and unite the lower half of the resulting diagram with the upper half of Fig. 11a to have the more detailed Fig. 12.

Fig. 12 is again the 4-geometry/intrinsic 2-geometry diagram of Fig. 5, in which the 'one-dimensional' absolute intrinsic metric space $\partial \hat{\rho}$ is curved relative to its projective straight line absolute proper

intrinsic metric space $\varnothing\rho'_{ab}$ along the horizontal and the absolute intrinsic metric time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ is curved relative to its projective straight line absolute proper intrinsic metric time dimension $\varnothing c_{sab}\varnothing t'_{ab}$ along the vertical. The new addition to Fig. 5 in Fig. 12 are the non-uniform absolute intrinsic 'static flow' speeds at every point along the curved absolute intrinsic metric space $\varnothing\hat{\rho}$ and along the curved absolute intrinsic metric time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$, where absolute intrinsic 'static flow' speeds at only two points P and Q along $\varnothing\hat{\rho}$ and at the symmetry-partner points P^0 and Q^0 along $\varnothing\hat{c}_s\varnothing\hat{t}$ are shown. The lower half of Fig. 12 is valid with respect to 3-observers in \mathbb{E}^3 , while the upper half is valid with respect to 1-observers in $c_s t'$.

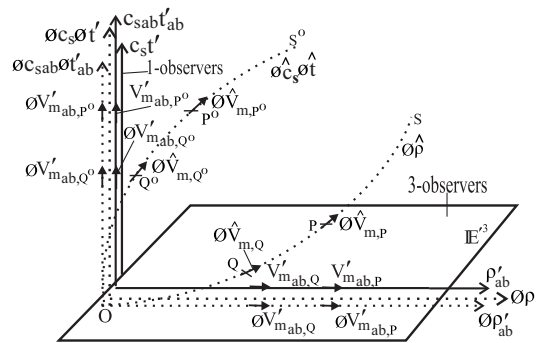


Fig. 12. Non-uniform absolute intrinsic 'static flow' speeds $\varnothing\hat{V}_m$ along the 'dimensions' of the curved absolute intrinsic static metric spacetime ($\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t}$) established by the symmetry-partner sources of a long-range absolute intrinsic metric force fields at positions S on and S^0 are projected invariantly as non-uniform absolute proper intrinsic 'static flow' speeds $\varnothing V'_{mab}$ ($= \varnothing\hat{V}$) along the 'dimensions' of the projective flat absolute proper intrinsic metric spacetime ($\varnothing\rho'_{ab}, \varnothing c_{sab}\varnothing t'_{ab}$) and are made manifested outwardly in non-uniform absolute proper static speeds V'_{mab} along the 'dimensions' of flat absolute proper metric spacetime ($\rho'_{ab}, c_{sab}t'_{ab}$) in our universe

The non-uniform absolute intrinsic 'static flow' speeds, $\varnothing\hat{V}_{m,P}$ and $\varnothing\hat{V}_{m,P^0}$, of the curved absolute intrinsic metric spacetime dimensions, $\varnothing\hat{c}_s\varnothing\hat{t}$ and $\varnothing\hat{\rho}$, derived geometrically above, but which are actually established along the curved $\varnothing\hat{\rho}$ and $\varnothing\hat{c}_s\varnothing\hat{t}$ by the source of an absolute intrinsic metric force field located at the origin (S, S^0) of the curved ($\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t}$) in Figs. 12, with maximum value $\varnothing\hat{c}_m$, is different from absolute intrinsic static geodesic flow speed $\varnothing\hat{V}_0$ (or $\varnothing\hat{V}_s$) with maximum value $\varnothing\hat{c}_s$, which appears in the notation of the absolute intrinsic static metric time dimension $\varnothing\hat{c}_s\varnothing\hat{t}$. The absolute intrinsic static geodesic flow speed $\varnothing\hat{V}_0$ (or $\varnothing\hat{V}_s$) has constant maximum value $\varnothing\hat{c}_s$ at every point of the curved $\varnothing\hat{c}_s\varnothing\hat{t}$, while $\varnothing\hat{V}_m$ established by the source of the metric force field has varying values less than the maximum value $\varnothing\hat{c}_m$ along the curved $\varnothing\hat{\rho}$ and $\varnothing\hat{c}_s\varnothing\hat{t}$ in Fig. 12.

As illustrated in Fig. 12, the absolute intrinsic 'static flow speeds, ' $\varnothing\hat{V}_{m,Q}$ and $\varnothing\hat{V}_{m,P}$, along the curved absolute intrinsic metric space $\varnothing\hat{\rho}$ are invariantly projected as absolute proper intrinsic 'static flow' speeds, $\varnothing V'_{mab,Q}$ and $\varnothing V'_{mab,P}$, into the straight line absolute proper intrinsic metric space $\varnothing\rho'_{ab}$ along the horizontal, with respect to 3-observers in \mathbb{E}^3 . The absolute intrinsic 'static flow' speeds, $\varnothing\hat{V}_{m,Q^0}$ and $\varnothing\hat{V}_{m,P^0}$, along the curved absolute intrinsic time dimension $\varnothing\hat{c}_s\varnothing\hat{t}$ are likewise invariantly projected as absolute proper intrinsic 'static flow' speeds, $\varnothing V'_{mab,Q^0}$ and $\varnothing V'_{mab,P^0}$, into the absolute proper intrinsic metric time dimension $\varnothing c_{sab}\varnothing t'_{ab}$ along the vertical, with respect to all 1-observers in $c_s t'$. The projective absolute proper intrinsic 'static flow' speeds, $\varnothing V'_{mab,Q}$ and $\varnothing V'_{mab,P}$, along $\varnothing\rho'_{ab}$ are then made manifested in absolute proper 'static flow' speeds, $V'_{mab,Q}$ and $V'_{mab,P}$, in the relative proper Euclidean 3-space \mathbb{E}^3 , just as the projective absolute proper intrinsic 'static flow' speeds, $\varnothing V'_{mab,Q^0}$ and $\varnothing V'_{mab,P^0}$, along $\varnothing c_{sab}\varnothing t'_{ab}$ are made manifested in absolute proper 'static

flow' speeds, V'_{mab,Q^0} and V'_{mab,P^0} , along the relative proper time dimension $c_s t'$, as illustrated in Fig. 12.

The prime label on the absolute proper intrinsic 'static flow' speeds, $\varnothing V'_{mab,Q}$, $\varnothing V'_{mab,P}$, $\varnothing V'_{mab,Q^0}$ and $\varnothing V'_{mab,P^0}$, makes them proper intrinsic speeds, while the subscript "ab" label makes them absolute intrinsic speeds. The absolute proper intrinsic 'static flow' speeds, $\varnothing V'_{mab,Q}$ and $\varnothing V'_{mab,P}$, are projected into the projective absolute proper intrinsic metric space dimension $\varnothing \rho'_{ab}$ that is imperceptibly embedded in the relative proper intrinsic metric space $\varnothing \rho'$ along the horizontal. The absolute proper intrinsic 'static flow' speeds, $\varnothing V'_{mab,Q^0}$ and $\varnothing V'_{mab,P^0}$, are likewise projected into the projective absolute proper intrinsic metric time dimension $\varnothing c_{sab} \varnothing t'_{ab}$ that is imperceptibly embedded in the relative proper intrinsic metric time dimension $\varnothing c_s \varnothing t'$ along the vertical in Fig. 12.

The projective absolute proper intrinsic 'static flow' speeds, $\varnothing V'_{mab,Q}$ and $\varnothing V'_{mab,P}$, and their maximum value $\varnothing c_{mab}$, in the projective straight line, $\varnothing \rho'_{ab}$ and $\varnothing c_{sab} \varnothing t'_{ab}$, are static (or non-dynamical) like, $\varnothing \hat{V}_{m,Q}$, $\varnothing \hat{V}_{m,P}$ and $\varnothing \hat{c}_m$, along the curved $\varnothing \hat{\rho}$ and $\varnothing \hat{c}_s \varnothing \hat{t}$ that project them.

The fact that absolute intrinsic 'static flow' speeds along the curved absolute intrinsic space $\varnothing \hat{\rho}$ and curved absolute intrinsic time dimension $\varnothing \hat{c}_s \varnothing \hat{t}$, are invariantly projected into the absolute proper intrinsic metric space $\varnothing \rho'_{ab}$ and absolute proper intrinsic metric time dimension $\varnothing c_{sab} \varnothing t'_{ab}$, as absolute proper intrinsic 'static flow' speeds, $\varnothing V'_{mab,Q}$, $\varnothing V'_{mab,P}$, $\varnothing V'_{mab,Q^0}$; $\varnothing V'_{mab,P^0}$ and $\varnothing c_{mab}$, in Fig. 12, in the context of absolute intrinsic Riemannian spacetime geometry (or in the context of the absolute intrinsic metric phenomena associated with absolute intrinsic Riemannian spacetime geometry), shall be stated as the following invariance

$$\varnothing V'_{mab} = \varnothing \hat{V}_m, \quad (83a)$$

hence

$$V'_{mab} = \hat{V}_m, \quad (83b)$$

where Eq. (83a) has been written at an arbitrary point along the curved $\varnothing \hat{\rho}$ and its symmetry-partner point along the curved $\varnothing \hat{c}_s \varnothing \hat{t}$ and Eq. (83b) has been written at the corresponding point in \mathbb{E}^{13} and its symmetry-partner point along $c_s t'$.

Let us re-write Eqs. (79) and (80), while letting, $\varnothing \hat{\psi}_{P^0} = \varnothing \hat{\psi}_P \equiv \varnothing \hat{\psi}$, and $\varnothing \hat{V}_{m,P^0} = \varnothing \hat{V}_{m,P} \equiv \varnothing \hat{V}_m$ in those equations as the following singular equation, which is valid at arbitrary symmetry-partner points along both $\varnothing \hat{\rho}$ and $\varnothing \hat{c}_s \varnothing \hat{t}$

$$\sin \varnothing \hat{\psi} = \varnothing \hat{V}_m / \varnothing \hat{c}_m. \quad (84a)$$

The relation for the identical absolute intrinsic curvature parameters $\varnothing \hat{k}$ at the arbitrary point P along the curved absolute intrinsic space $\varnothing \hat{\rho}$ with respect to 3-observers in \mathbb{E}^{13} and at the symmetry-partner point along the curved absolute intrinsic metric time 'dimension' $\varnothing \hat{c}_s \varnothing \hat{t}$ with respect to 1-observers in $c_s t'$, has been related to the absolute intrinsic angle $\varnothing \hat{\psi}$ in sub-section 1.1 of [2] as

$$\sin \varnothing \hat{\psi} = \varnothing \hat{k}. \quad (84b)$$

The absolute intrinsic curvature parameter at an arbitrary point P along the curved $\varnothing \hat{\rho}$ and at the symmetry-partner point P^0 along the curved $\varnothing \hat{c}_s \varnothing \hat{t}$ is therefore related to the absolute intrinsic 'static flow' speed at those points from Eqs. (84a) and (84b) as

$$\varnothing \hat{k} = \varnothing \hat{V}_m / \varnothing \hat{c}_m. \quad (84c)$$

The absolute intrinsic metric tensor and absolute intrinsic Ricci tensor without star label, given in terms of absolute intrinsic curvature parameter as Eqs. (64) and (68), in the case of one absolute intrinsic Riemannian metric spacetime, that is, in the case of a singular curved absolute intrinsic

metric spacetime, can then be written in terms of the absolute intrinsic 'static flow' speeds, $\varnothing\hat{V}_m$ and $\varnothing\hat{c}_m$, respectively as

$$\varnothing\hat{g}_{ij} = \begin{pmatrix} 1 - \varnothing\hat{V}_m^2/\varnothing\hat{c}_m^2 & 0 \\ 0 & -\frac{1}{1 - \varnothing\hat{V}_m^2/\varnothing\hat{c}_m^2} \end{pmatrix} \quad (85)$$

and

$$\varnothing\hat{R}_{ij} = \begin{pmatrix} -\varnothing\hat{V}_m^2/\varnothing\hat{c}_m^2 & 0 \\ 0 & -\frac{\varnothing\hat{V}_m^2/\varnothing\hat{c}_m^2}{1 - \varnothing\hat{V}_m^2/\varnothing\hat{c}_m^2} \end{pmatrix}. \quad (86)$$

The absolute intrinsic line element (62) likewise becomes the following in terms of the absolute intrinsic 'static flow' speeds $\varnothing\hat{V}_m$ and $\varnothing\hat{c}_m$

$$d\varnothing\hat{s}^2 = (1 - \varnothing\hat{V}_m^2/\varnothing\hat{c}_m^2)\varnothing\hat{c}_m^2 d\varnothing\hat{t}^2 - \frac{d\varnothing\hat{\rho}^2}{1 - \varnothing\hat{V}_m^2/\varnothing\hat{c}_m^2}. \quad (87)$$

The resultant absolute intrinsic metric tensor, resultant absolute intrinsic Ricci tensor and resultant absolute intrinsic line element (73), (76) and (78), in a situation where two absolute intrinsic Riemannian metric spacetimes co-exist, become the following in terms of the absolute intrinsic 'static flow' speeds $\varnothing\hat{V}_m$, $\varnothing\hat{V}'_m$ and $\varnothing\hat{c}_m$

$$\varnothing\hat{g}_{ij} = \begin{pmatrix} 1 - \frac{\varnothing\hat{V}'_m{}^2}{\varnothing\hat{c}_m^2} - \frac{\varnothing\hat{V}_m^2}{\varnothing\hat{c}_m^2} & 0 \\ 0 & -\frac{1}{1 - \frac{\varnothing\hat{V}'_m{}^2}{\varnothing\hat{c}_m^2} - \frac{\varnothing\hat{V}_m^2}{\varnothing\hat{c}_m^2}} \end{pmatrix} \quad (88)$$

$$\varnothing\hat{R}_{ij} = \begin{pmatrix} -\frac{\varnothing\hat{V}'_m{}^2}{\varnothing\hat{c}_m^2} - \frac{\varnothing\hat{V}_m^2}{\varnothing\hat{c}_m^2} & 0 \\ 0 & -\frac{\frac{\varnothing\hat{V}'_m{}^2}{\varnothing\hat{c}_m^2} + \frac{\varnothing\hat{V}_m^2}{\varnothing\hat{c}_m^2}}{1 - \frac{\varnothing\hat{V}'_m{}^2}{\varnothing\hat{c}_m^2} - \frac{\varnothing\hat{V}_m^2}{\varnothing\hat{c}_m^2}} \end{pmatrix} \quad (89)$$

and

$$(d\varnothing\hat{s})^2 = \left(1 - \frac{\varnothing\hat{V}'_m{}^2}{\varnothing\hat{c}_m^2} - \frac{\varnothing\hat{V}_m^2}{\varnothing\hat{c}_m^2}\right)\varnothing\hat{c}_m^2 d\varnothing\hat{t}^2 - \frac{d\varnothing\hat{\rho}^2}{1 - \frac{\varnothing\hat{V}'_m{}^2}{\varnothing\hat{c}_m^2} - \frac{\varnothing\hat{V}_m^2}{\varnothing\hat{c}_m^2}}. \quad (90)$$

Extension of Eqs. (88) - (90) to situations where a larger number of absolute intrinsic Riemannian metric spacetimes (or curved 'two-dimensional' absolute intrinsic metric spacetimes) co-exist (or are superposed) is straight forward.

The absolute intrinsic curvature parameter $\varnothing\hat{k}$ is a geometrical parameter, as follows from its derivation in sub-section 1.1 of part two of this paper [2]. The non-uniform static absolute intrinsic 'static flow' speed $S\varnothing\hat{V}_m$ along the curved absolute intrinsic space $\varnothing\hat{\rho}$ and curved absolute intrinsic time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ in

Fig. 12, which is related to the non-uniform absolute intrinsic curvature parameters $\varnothing\hat{k}$ of the curved $\varnothing\hat{\rho}$ and curved $\varnothing\hat{c}_s\varnothing\hat{t}$, by Eq. (84c), is likewise an absolute intrinsic geometrical parameter. This is so, because the definition, $\varnothing\hat{V}_m = d\varnothing\hat{\rho}/d\varnothing\hat{t}$, in Eqs. (79) and (80), follows from the geometry of Figs. 10a and 10b, without relation to the absolute intrinsic parameters of the absolute intrinsic metric force field that establishes absolute intrinsic Riemann geometry. The absolute intrinsic geometrical parameters, $\varnothing\hat{V}_m$ and $\varnothing\hat{c}_m$, or $\varnothing\hat{k}$, which appear in the absolute intrinsic metric tensor, absolute intrinsic

Ricci tensor and absolute intrinsic line element, in absolute intrinsic Riemannian spacetime geometry, shall be related to the absolute intrinsic parameters of the absolute intrinsic metric force field that give rise to curved 'two-dimensional' absolute intrinsic metric spacetime $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$ elsewhere.

The explanation of the evolution of the curved absolute intrinsic metric space $\varnothing\hat{\rho}$ and curved absolute intrinsic metric time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ in Fig. 12 or Fig. 5, from the reference geometry of Fig. 7, which follows from the validity of Eqs. (81) and (82) at every point along the curved $\varnothing\hat{\rho}$ and $\varnothing\hat{c}_s\varnothing\hat{t}$ in Fig. 12 is that non-uniform absolute intrinsic 'static flow' speeds $\varnothing\hat{V}_m$ are identically established along the straight line absolute intrinsic metric space $\varnothing\hat{\rho}$ and straight line absolute intrinsic metric time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ from a point (S, S^0) on the flat 'four-dimensional' absolute spacetime $(\mathbb{I}\hat{\mathbb{E}}^3, \hat{c}_s\hat{t})$ in Fig. 7. Then the geometry of Fig. 12 evolves as a consequence, since (84a) must be satisfied at every point along $\varnothing\hat{\rho}$ and $\varnothing\hat{c}_s\varnothing\hat{t}$. The mechanism by which this is achieved in nature requires explanation to be given elsewhere.

The geometry of Fig. 12 will evolve from Fig. 7, for instance, if the spherical source of a long-range absolute metric force field (such as the source of an absolute gravitational field) located at a point S in the absolute space $\mathbb{I}\hat{\mathbb{E}}^3$ of our universe in Fig. 7, establishes non-uniform absolute 'static flow' speeds \hat{V}_m along every radial direction from its center in all its finite neighborhood in $\mathbb{I}\hat{\mathbb{E}}^3$ and the source of absolute intrinsic metric force field in the absolute intrinsic space $\varnothing\hat{\rho}$, underlying the source of absolute metric force field in $\mathbb{I}\hat{\mathbb{E}}^3$, establishes non-uniform absolute intrinsic 'static flow' speeds $\varnothing\hat{V}_m$ along the straight line absolute intrinsic metric space $\varnothing\hat{\rho}$ in all its finite neighborhood in Fig. 7. This will give rise to the curved $\varnothing\hat{\rho}$ and its projective straight line absolute proper intrinsic metric space $\varnothing\rho'_{ab}$ along the horizontal in our universe as illustrated in Fig. 12.

The identical symmetry-partner source of long-range absolute metric force field in the flat absolute space $\mathbb{I}\hat{\mathbb{E}}^{03}$ and identical source of long-range absolute intrinsic metric force field

in straight line absolute intrinsic space $\varnothing\rho^0$, in the geometry in the positive time-universe corresponds to that of Fig. 7 in our universe, will give rise to curved absolute intrinsic metric space $\varnothing\hat{\rho}^0$ that projects straight line absolute proper intrinsic metric space $\varnothing\rho^0_{ab}$ along the vertical (as illustrated in Fig. 3) in the positive time-universe. This corresponds to curved absolute intrinsic time 'dimension' $\varnothing\hat{c}_s\varnothing\hat{t}$ and its projective absolute proper intrinsic time dimension $\varnothing c_{sab}\varnothing t'_{ab}$ of our universe along the vertical in Fig. 4 or Fig. 12.

4 CONCLUSION

This third part of this paper is the conclusion of the first stage of evolutions of metric spacetime and intrinsic metric spacetimes in long-range metric force fields and the derivation of the associated absolute intrinsic Riemannian spacetime geometry, started in the first two parts. The first stage can be described as numerical evolution, because the entries of the absolute intrinsic metric tensor $\varnothing\hat{g}_{ik}$ and absolute intrinsic Ricci tensor $\varnothing\hat{R}_{ik}$ involved are numbers for discretized elementary segments of the curved 'two-dimensional' absolute intrinsic metric spacetime $(\varnothing\hat{\rho}, \varnothing\hat{c}_s\varnothing\hat{t})$. Moreover the two absolute intrinsic metric tensor equations derived for $\varnothing\hat{g}_{ik}$ and $\varnothing\hat{R}_{ik}$ are possible of algebraic solution. The final fourth part shall be devoted to the second (and final) stage of evolutions of relative metric spacetime and relative intrinsic metric spacetimes in long-range metric force fields and the development of the associated local Lorentzian metric spacetime/intrinsic local Lorentzian metric spacetime geometry.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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