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Analysis of an Eco-Epidemiological Model Considering Effect of Harvesting and Prey Refuge

Aihua Kang

College of General Education, Shanxi College of Technology, Shuozhou, China
Email: *kangaihua1982@163.com

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Abstract

In this work, we analyze an eco-epidemiological model with the disease in the prey, considering a constant proportion of harvesting of either species or a prey refuge. The positive invariant set, the conditions of existence, and locally asymptotically stability of the equilibria are studied using the stability theory of ordinary differential equation. The global stability of border equilibria by constructing Lyapunov functions and permanence of the system by comparison theoremare proved. The numerical simulation further proved the correctness of the theoretical analysis. The result indicates that overfishing would lead to population extinction and a reasonable fishing strategy should keep the coexistence of populations.

Keywords

Refuge, Harvesting, Eco-Epidemiological Model, Permanence

1. Introduction

Eco-epidemiology is a new offshoot of biological mathematics which considers the dynamics of infectious disease spreading among the ecosystem, which has already been researched in a lot of articles [1] [2]. They discussed the predator-prey system with the disease among prey population [3], predator population [4], both predators and prey populations.

The effect of refuges is vital on the ecosystem population, and research on the dynamic behaviors of incorporating prey refuges has become the main topic during the last decade [5] [6] [7]. A prey refuge is taken into account from two types. One type is a fixed proportion of prey refuge: $(1-\nu)x$, where $\nu \in (0,1)$, another type is a fixed quantity of prey refuge: $x-\nu$. A prey refuge may lead to population extinction or system's fluctuations. For example, a destabilization phenomenon appears because of prey refuge in an eco-epidemiological system [8]. Choosing different prey refuge decides whether undergoes Hopf bifurcation [9].

Reasonable harvesting policy also has been done for a long time of research, which is one of the interesting and main problems in economics and ecology [10]. For example, a predator-prey model with constant proportion of prey harvesting is studied [11]. Biwen Li *et al.* [12] researched a predator-prey model taking into account nonselective harvesting. The complex behavior of an eco-epidemiological model considering prey catching was investigated by S.N. Raw and Barkha Tiwari [13].

Based on the previous research, we formulate an eco-epidemiological model, considering a constant proportion of harvesting of either species or a prey refuge. The first purpose of the paper is to study the complex behavior (stability, branches and permanence) of the model. The second purpose is to study the impact of harvesting of different populations on the extinction of disease.

The organization of the paper is as follows: In Section 2, a prey-predator model with the disease in the prey, considering a constant proportion of harvesting of either species or prey refuge is built. The conditions of stability of equilibria and the sufficient condition for the permanence of the model are obtained in Section 3. Furthermore, in Section 4, we analyze the impact of harvesting of different populations. The correctness of the theoretical analysis is verified by numerical simulation in Section 5. And our findings are briefly discussed in Section 6.

2. The Mathematical Model

Assumptions: 1) In the existence of infection, prey population is separated into two parts—the susceptible prey $S(t)$, the infected prey $I(t)$. The whole prey population grows in terms of logistic growth with carrying capacity k and an intrinsic birth rate r, namely

$$\dot{N}(t) = rN\left[1-\frac{N}{k}\right], \text{ where } N(t) = S(t)+I(t).$$

2) The disease is only spreading among prey populations and not inherited. The infected population doesn't recover. Because of infection, infected preys are caught easily. The predator only consumes infected prey according to the Holling III functional response. A conversion rate of consumed prey is η.

3) Only the susceptible preys have reproductive capacity. The death rates of predator and infected prey are d, c, respectively.

4) $m(0<m<1)$ is a fixed rate of infected prey using refuges.

5) $E_1>0$, $E_2>0$, $E_3>0$ express the harvesting capabilities for the predator, susceptible prey, the infected prey, respectively. q_1E_1S, q_2E_2I, and q_3E_3Y, represent the catch of the respective species, where q_1, q_2, q_3 represent the catch efforts coefficients of the predator, the susceptible prey and the infected

prey respectively. All the parameters mentioned are supposed to be positive.

We construct the following model.

$$\begin{cases} \dot{S}(t) = rS\left(1-\dfrac{S+I}{k}\right) - \beta SI - q_1E_1S \\ \dot{I}(t) = \beta SI - \dfrac{(1-m)^2 I^2 Y}{b+(1-m)^2 I^2} - q_2E_2I - cI \\ \dot{Y}(t) = -dY + \dfrac{\eta(1-m)^2 I^2 Y}{b+(1-m)^2 I^2} - q_3E_3Y \end{cases} \tag{2.1}$$

Theorem 2.1 All solutions of system (2.1) satisfying initial conditions in R^3 are ultimately bounded.

Proof:Let $\varpi = \eta S + \eta I + Y$, then

$$\begin{aligned} \frac{\mathrm{d}\varpi}{\mathrm{d}t} &= \eta\frac{\mathrm{d}S}{\mathrm{d}t} + \eta\frac{\mathrm{d}I}{\mathrm{d}t} + \frac{\mathrm{d}Y}{\mathrm{d}t} \\ &= rS\left[1-\frac{S}{k}\right] - \frac{\eta r}{k}SI - \eta q_1E_1S - (\eta c + \eta q_2E_2)I - (d+q_3E_3)Y \\ &\le \eta rS - \eta q_1E_1S - (\eta c+\eta q_2E_2)I - (d+q_3E_3)Y \\ &\le \eta rk - \left[\eta q_1E_1S + (\eta c+\eta q_2E_2)I + (d+q_3E_3)Y\right] \\ &\le \eta rk - \theta\varpi \end{aligned}$$

where $0<\theta<\min\{\eta q_1E_1, (\eta c+\eta q_2E_2), (d+q_3E_3)\}$ then

$$\frac{\mathrm{d}\varpi}{\mathrm{d}t} + \theta\varpi \le \mu rk$$

The linear differential inequality is solved to obtain the result.

$$0<\varpi(S,I,Y)<\frac{\eta rk}{\theta}\left(1-\mathrm{e}^{-\theta t}\right)+\varpi\left(S(0),I(0),Y(0)\right)\mathrm{e}^{-\theta t}$$

If $t\to\infty$, $0<\varpi<\dfrac{\eta rk}{\theta}$

The model (2.1) of all solutions starting in $\left(S(0),I(0),Y(0)\right)\in R_+^3$ remain in R^3 for all $t\ge 0$. Thus, all trajectories of system (2.1) will enter the area: $M=\left\{(S,I,Y)\in R_+^3 : \varpi = \dfrac{\eta rk}{\theta}+\zeta, \zeta>0\right\}$

3. Stability Analysis

3.1. Existing of Equilibrium Points

The system (2.1) has several equilibria:

1) The zero equilibrium $E_a=(0,0,0)$,

2) The axial equilibrium $E_b=\left(\dfrac{k(r-q_1E_1)}{r},0,0\right)$,

3) The boundary equilibrium $E_c=(S_2,I_2,0)$, and $S_2=\dfrac{c+q_2E_2}{\beta}$,

$I_2=\dfrac{rk\beta - rc - rq_2E_2 - k\beta q_1E_1}{\beta(r+k\beta)}$,

4) The positive equilibrium $E_* = (S_*, I_*, P_*)$ where

$$S_* = \frac{rk - (r + k\beta) I_* - kq_1E_1}{r}, \quad I_* = \sqrt{\frac{b(d + q_3E_3)}{(\eta - d - q_3E_3)(1-m)^2}},$$

$$Y_* = \frac{(\beta s_* - c - q_2E_2)\left(b + (1-m)^2 I_*^2\right)}{(1-m)^2 I_*^2},$$

If $r > q_1E_1$, the equilibrium point E_b exists;

If $rk\beta > rc + rq_2E_2 + k\beta q_1E_1$, the equilibrium points E_c exists;

If $\eta - d - q_3E_3 > 0$, $rk - (r + k\beta) I_* - kq_1E_1 > 0$, and $\beta s_* - c - q_2E_2 > 0$, the equilibrium point E_* exists.

3.2. Locally Stability

Let $\bar{E} = (\bar{S}, \bar{I}, \bar{Y})$ is an arbitrary equilibrium point of the model (2.1), the Jacobian matrix at arbitrary equilibrium point is

$$\begin{vmatrix} r - \frac{2r}{k}\bar{S} - \frac{r}{k}\bar{I} - \beta\bar{I} - q_1E_1 & -\frac{r}{k}\bar{S} - \beta\bar{S} & 0 \\ \beta\bar{I} & \beta\bar{S} - \frac{2b(1-m)^2 \bar{I}\bar{Y}}{\left[b + (1-m)^2 I^2\right]^2} - (c + q_2E_2) & -\frac{(1-m)^2 I^2}{b + (1-m)^2 \bar{I}^2} \\ 0 & \frac{2\eta b(1-m)^2 \bar{I}\bar{Y}}{\left[b + (1-m)^2 \bar{I}^2\right]^2} & -d + \frac{\eta(1-m)^2 \bar{I}^2}{b + (1-m)^2 \bar{I}^2} - q_3E_3 \end{vmatrix} \tag{3.1}$$

By calculating the positive or negative of the real part of the eigenvalue, the local asymptotic stability of each equilibrium point can be judged. The details are as follows:

1) $\lambda_1 = r - q_1E_1$, $\lambda_2 = -(c + q_2E_2)$, $\lambda_3 = -(d + q_3E_3)$ are three eigenvalues of the Jacobian matrix at E_a. The conditions of local asymptotical stability of E_a are $r < q_1E_1$, $\lambda_1 < 0$

2) $\lambda_1 = -(r - q_1E_1)$, $\lambda_2 = \frac{rk\beta - rc - rq_2E_2 - k\beta q_1E_1}{r}$, $\lambda_3 = -(d + q_3E_3)$ are three eigenvalues of the Jacobian matrix at E_b. If $rk\beta < rc + rq_2E_2 + k\beta q_1E_1$, $\lambda_2 < 0$, E_b is locally asymptotically stable

3) Concerning E_c, its characteristic equation is

$$\mu^3 + A_1\mu^2 + A_2\mu + A_3 = 0$$

where $A_1 = -(a_{11} + a_{33})$, $A_2 = a_{11}a_{33} - a_{12}a_{21}$, $A_3 = a_{12}a_{21}a_{33}$ and $a_{11} = -\frac{rS_2}{k}$, $a_{12} = -\left(\frac{r}{k} - \beta\right)S_2$, $a_{21} = \beta I_2$, $a_{33} = -d + \frac{\eta(1-m)^2 I_2^2}{b + (1-m)^2 I_2^2} - q_3E_3$.

If $\eta - d - q_3E_3 < 0$, $a_{33} < 0$, And $A_1 > 0$, $A_3 > 0$, $A_1A_2 - A_3 > 0$. According to the Routh-Hurwitz rule, all eigenvalues of the Jacobian matrix at E_c are negative. The local asymptotical stability of E_c is obtained.

4) Concerning E_*, its characteristic equation is

$$\mu^3 + B_1\mu^2 + B_2\mu + B_3 = 0$$

where $B_1 = 2\frac{r}{k}S_* + \beta S_*$,

$$B_2 = -\frac{r}{k}S_*\left[\frac{(1-m)^2 I_*^2 Y_*}{b+(1-m)^2 I_*^2} - \frac{2b(1-m)^2 I_* Y_*}{\left(b+(1-m)^2 I_*^2\right)^2}\right]$$
$$+\beta\left(\frac{r}{k}+\beta\right)S_* I_* + \frac{2b\eta(1-m)^4 I_*^3 Y_*}{\left(b+(1-m)^2 I_*^2\right)^3}$$

$$B_3 = \frac{2ar\eta(1-m)^4 S_* I_*^3 Y_*}{k\left(b+(1-m)^2 I_*^2\right)^3}.$$

Obviously, $H_1 = B_1 > 0$, $B_3 > 0$.

If $\eta^2 b(d+q_3E_3) < 4(\eta-d-q_3E_3)^3(1-m)^2$, $H_2 = B_1B_2 - B_3 > 0$, so $H_3 = B_3H_2 > 0$.

In according with the Routh-Hurwitz rule, all eigenvalues of the Jacobian matrix at E_* are negative. The local asymptotical stability of E_* is obtained.

3.3. Globally Asymptotically Stable

The global asymptotically stability of each equilibrium point is judged by constructing the Lyapunov function.

Theorem 3.1 The condition of global asymptotical stability of E_a is $rk - r - kq_1E_1 < 0$ on plane $\Gamma = \{(S,I,Y): S>0, I>0, Y>0\}$.

Proof: The Lyapunov function $V_1 = S + I + \frac{Y}{\mu}$ is constructed, then we have

$$\frac{\mathrm{d}V_1}{\mathrm{d}t} = r\left(1-\frac{S}{k}\right)S - q_1E_1S - \frac{r}{k}SI - (c+q_2E_2)I - \frac{1}{\theta}(d+q_3E_3)Y$$
$$\le \left(r-\frac{r}{k}-q_1E_1\right)S - (c+q_2E_2)I - \frac{1}{\theta}(d+q_3E_3)Y$$

If $rk - r - kq_1E_1 < 0$, $\frac{\mathrm{d}V}{\mathrm{d}t} \le 0$ And,

$$D_1 = \left\{(S,I,Y)\in M \mid \frac{\mathrm{d}V_1}{\mathrm{d}t} = 0\right\} = \{(S,I,Y) = (0,0,0)\}.$$

So, the global asymptotical stability of E_a is proved.

Theorem 3.2 The condition of global asymptotical stability of E_b is $\beta k - c - q_2E_2 < 0$ on plane $\Gamma = \{(S,I,Y): S>0, I>0, Y>0\}$.

Proof: A Lyapunov function $V_2 = \eta I + Y$ is constructed, then we obtain that

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \eta(\beta S - c - q_2E_2)I - (d+q_3E_3)Y$$
$$\le \eta(\beta k - c - q_2E_2)I - (d+q_3E_3)Y$$

Hence, if $\beta k - c - q_2E_2 < 0$, $\frac{\mathrm{d}V}{\mathrm{d}t} \le 0$.

and

$$D_2=\left\{(S,I,Y)\in M \mid \frac{\mathrm{d}V}{\mathrm{d}t}=0\right\}=\{(S,I,Y)=(S,0,0)\}$$

the maximum invariable set of system (2.1) is $\Gamma_1=\{I=0,Y=0\}$.

So the limit equation of system (2.1) is

$$\frac{\mathrm{d}S}{\mathrm{d}t}=rS\left(1-\frac{S}{k}\right)-q_1E_1S$$

Obviously, the global asymptotic stability of E_b is obtained.

Theorem 3.3. The condition of global asymptotical stability of E_c is $d+q_3E_3-\eta>0$ on plane $\Gamma=\{(S,I,Y):S>0,I>0,Y>0\}$

Similarly, The Lyapunov function $V_3=Y$ is constructed,

$$\begin{aligned}\frac{\mathrm{d}V_3}{\mathrm{d}t}&=-\left[d-\frac{\eta(1-m)^2I^2}{b+(1-m)^2I^2}+q_3E_3\right]Y\\&=-\frac{(d+q_3E_3)b+(d+q_3E_3-\eta)(1-m)^2I^2}{b+(1-m)^2I^2}Y\end{aligned}$$

If $d+q_3E_3-\eta>0$, obviously $\frac{\mathrm{d}V}{\mathrm{d}t}\le 0$, and

$$D_3=\left\{(S,I,Y)\in B \mid \frac{\mathrm{d}V}{\mathrm{d}t}=0\right\}=\{(S,I,Y)=(S,I,0)\}$$

the maximum invariable set of system (2.1) is $\Sigma=\{Y=0\}$. So, the global asymptotical stability of E_c is obtained.

3.4. Branching Phenomenon

Theorem 3.4. If the following conditions are true, Hopf bifurcation occurs at the equilibrium point of E_*.

i) If $E_2=E_{2*}$, $B_1>0$, $B_3>0$;

ii) If $E=E_{2*}$, $B_1B_2-B_3=0$, the eigenvalue of E_* has a pair of double virtual roots;

iii) $Re\left[\frac{\mathrm{d}\mu_j}{\mathrm{d}E_2}\right]_{E_2=E_{2*}}\neq 0$, ($j=1,2,3$).

Proof:

According to the previous characteristic equation of E_*, if $B_1B_2-B_3=0$, obtaining $\theta=\theta_*$, the characteristic equation of E_* can be written as

$$(\mu^2+B_2)(\mu+B_1)=0$$

The three eigenvalues are $i\sqrt{B_2},-i\sqrt{B_2},-B_1$.

Simultaneous derivation of E_2 on both sides of the equation, obtaining

$$3\mu^2\frac{\mathrm{d}\mu}{\mathrm{d}E_2}+\frac{\mathrm{d}B_1}{\mathrm{d}E_2}\mu^2+2B_1\mu\frac{\mathrm{d}\mu}{\mathrm{d}E_2}+\frac{\mathrm{d}B_2}{\mathrm{d}E_2}\mu+B_2\frac{\mathrm{d}\mu}{\mathrm{d}E_2}+\frac{\mathrm{d}B_3}{\mathrm{d}E_2}=0$$

Namely,

$$\left.\frac{\mathrm{d}\mu}{\mathrm{d}E_2}\right|_{E_2=E_{2*}} = -\left[\frac{\frac{\mathrm{d}Q_1}{\mathrm{d}E_2}\mu^2+\frac{\mathrm{d}Q_2}{\mathrm{d}E_2}\mu+\frac{\mathrm{d}B_3}{\mathrm{d}E_2}}{3\mu^2+2B_1\mu+Q_2}\right]_{\mu=i\sqrt{B_2}}$$

$$= -\left[\frac{\frac{\mathrm{d}(B_1B_2-B_3)}{\mathrm{d}\mu}}{2\left(B_1^2+B_2\right)}\right]_{E_2=E_{2*}} + i\left[\frac{\sqrt{B_2}\frac{\mathrm{d}B_2}{\mathrm{d}E_2}}{2B_2}-\frac{\sqrt{B_1}\frac{\mathrm{d}(B_1B_2-B_3)}{\mathrm{d}E_2}}{2B_2\left(B_1^2+B_2\right)}\right]_{E_2=E_{2*}}$$

In other words, $Re\left[\frac{\mathrm{d}\mu}{\mathrm{d}E_2}\right]_{E_2=E_{2*}} = -\left[\frac{\frac{\mathrm{d}(B_1B_2-B_3)}{\mathrm{d}\mu}}{2\left(B_1^2+B_2\right)}\right]_{E_2=E_{2*}} \neq 0$

3.5. Permanence

Theorem 3.5. If $rk\beta < rc + rq_2E_2 + k\beta q_1E_1$ and $\left(rk\beta - rc - rq_2E_2 - k\beta q_1E_1\right)^2\left(\eta - d + q_3E_3\right) > \left[\beta\left(r+k\beta\right)\right]^2\left(d+q_3E_3\right)b$, the system (2.1) is permanence.

Proof: For the sake of proving the permanence of system (2.1), a average Lyapunov function $V(S,I,Y) = S^{\alpha_1}I^{\alpha_2}Y^{\alpha_3}$ is constructed, where $\alpha_1,\alpha_2,\alpha_3$ is positive in the plan R^3,

$$\frac{V'}{V} = \alpha_1\left(r\left(1-\frac{S+I}{k}\right)-\beta I-q_1E_1\right)+\alpha_2\left(\beta S-c-\frac{(1-m)^2IY}{b+(1-m)^2I^2}-q_2E_2\right)$$

$$\alpha_3\left(-d+\frac{\eta(1-m)^2I^2}{a+(1-m)^2I^2}-q_3E_3\right) \equiv \Phi(S,I,Y)$$

The following conclusions need to be verified: $\Phi(S,I,Y) > 0$ for each boundary equilibrium point.

Let $\alpha_1 > \alpha_2\left(c+q_2E_2\right)+\alpha_3\left(d+q_3E_3\right)$, then

$$\Phi(E_0) = \alpha_1 r - \alpha_2\left(c+q_2E_2\right)-\alpha_3\left(d+q_3E_3\right)$$

$$\Phi(E_1) = \alpha_2\left[\beta\frac{k\left(r-q_1E_1\right)}{r}-q_2E_2\right]-\alpha_3\left(d+q_3E_3\right)$$

$$\Phi(E_2) = \alpha_3\left(-d+\frac{\mu(1-m)^2I_2^2}{b+(1-m)^2I_2^2}-q_3E_3\right)$$

Obviously, $\Phi(E_a) > 0$. If $rk\beta < rc+rq_2E_2+k\beta q_1E_1$, $\Phi(E_b) > 0$.

If $\left(rk\beta - rc - rq_2E_2 - k\beta q_1E_1\right)^2\left(\mu - d + q_3E_3\right) > \left[\beta\left(r+k\beta\right)\right]^2\left(d+q_3E_3\right)b$, $\Phi(E_c) > 0$.

4. The Impact of Harvesting

Three aspects are mainly discussed.

1) Taking into account the harvesting of susceptible prey E_1.

$$\frac{\partial S_*}{\partial E_1} = -\frac{kq_1}{r} < 0, \quad \frac{\partial I_*}{\partial E_1} = 0, \quad \frac{\partial Y_*}{\partial E_1} = 0$$

The increased the harvesting effort E_1 can decrease the quantity of susceptible prey and not change the quantity of infected prey and predator. If $E_1 < \frac{rk-r}{kq_1}$, the three population coexist. In other words, the disease exists. However, if $E_1 > \frac{r}{q_1}$, the prey and the predator all die out, which is the sufficiency condition of the globally asymptotically stable of E_a. If $\frac{rk-r}{kq_1} < E_1 < \frac{r}{q_1}$, the equilibrium point E_b exists, both infected prey and predator become extinct.

2) Taking into account the harvesting of infected prey E_2.

$$\frac{\partial S_*}{\partial E_2} = 0, \quad \frac{\partial I_*}{\partial E_2} = 0, \quad \frac{\partial Y_*}{\partial E_2} = -\frac{q_2}{(1-m)^2 I^2} < 0$$

Increasing of E_2 can decrease the quantity of predator population and not change the density of infected prey and uninfected prey. However, if $E_2 > \frac{k\beta - c}{q_2}$, the predator dies out, which is the condition of the globally asymptotically stable of E_b, which is so obvious, because the increasing of E_2 may result in the lack of food for predator.

3) The case is only considering the harvesting predator species.

$$\begin{aligned}\frac{\partial S_*}{\partial E_3} &= -\frac{(r+k\beta) bq_3\eta(1-m)^2}{2r(\eta-d-q_3E_3)^2(1-m)^2\sqrt{\frac{b(d+q_3E_3)}{(\eta-d-q_3E_3)(1-m)^2}}} \\ &\quad \times \frac{bq_3\eta(1-m)^2}{(\eta-d-q_3E_3)^2(1-m)^2} \\ &< 0\end{aligned}$$

$$\frac{\partial I_*}{\partial E_3} = \frac{bq_3\eta(1-m)^2}{2(\eta-d-q_3E_3)^2(1-m)^2\sqrt{\frac{b(d+q_3E_3)}{(\eta-d-q_3E_3)(1-m)^2}}} > 0,$$

$$\begin{aligned}\frac{\partial Y_*}{\partial E_2} &= -\frac{q_2}{(1-m)^2 I^2} \\ &= \frac{\beta\left[b+(1-m)^2 I_*^2\right]}{(1-m)^2 I_*^2}\frac{\partial S_*}{\partial E_3} - \frac{2b(1-m)^2 I_*(\beta S_* - c - q_3E_3)}{(1-m)^2 I_*^4}\frac{\partial I_*}{\partial E_3} \\ &< 0\end{aligned}$$

The increasing the harvesting effort E_3 can result in raising the quantity of infected prey and reducing the quantity of predator and susceptible prey. The condition of globally asymptotically stability of E_c is $E_3 > \frac{\eta - d}{q_3}$. In other words, the predator becomes extinct.

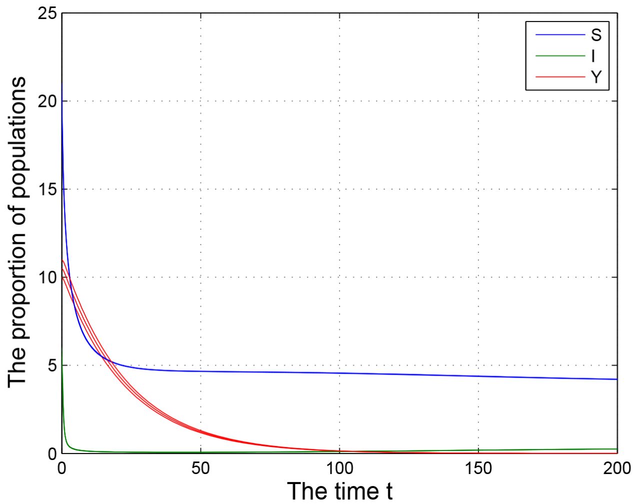
5. Number Simulations

Numerical simulation is used to further prove the correctness of theoretical analysis.

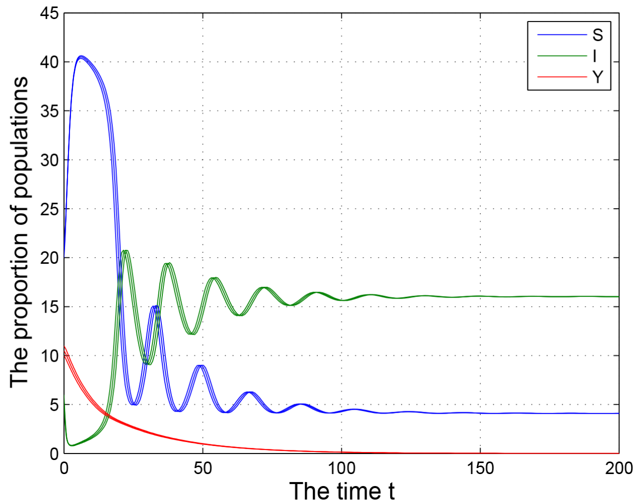
Let $r = 0.9$, $k = 48$, $\beta = 0.03$, $c = 0.15$, $m = 0.45$, $b = 0.4$, $d = 0.04$, $\eta = 0.036$. By assuming some parameters from the perspective of practical problems, we may observe the system (1.2) has the various results:

By changing different parametric values of q_1, E_1, q_2, E_2, q_3, E_3, we may observe the system (1.2) has the various results:

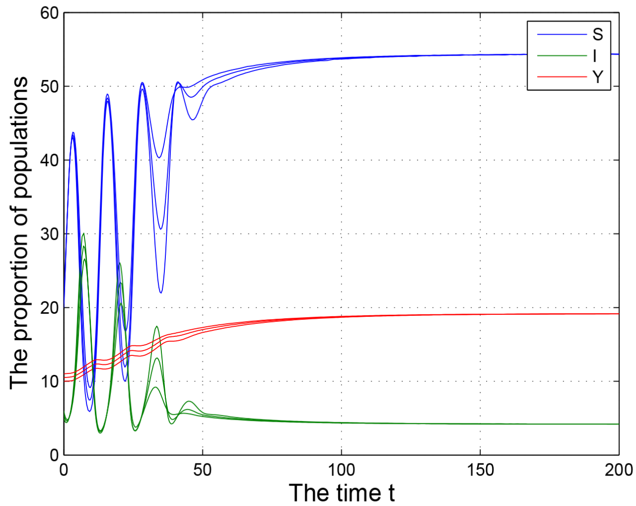
Along with q_1, E_1 increasing, the infected preys I and the predators Y are beginning to tend to 0, see **Figure 1(a)**. Along with q_3, E_3 increasing, the predators Y are beginning to tend to 0, see **Figure 1(b)**. But, along with q_2, E_2 changing, the stability of the system (2.1) undergoes changes: from state (see **Figure 1(c)**)



(a)

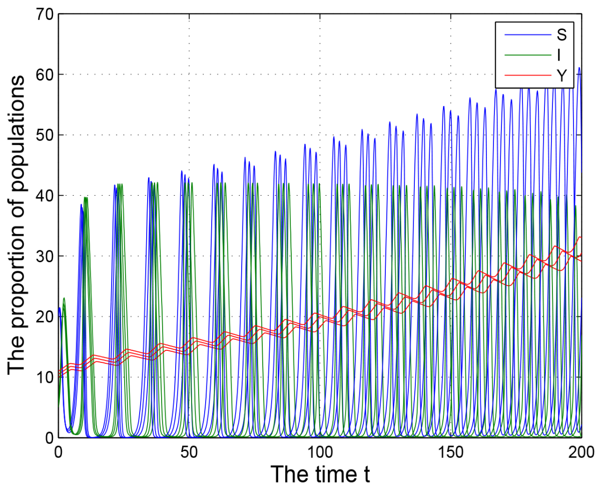


(b)

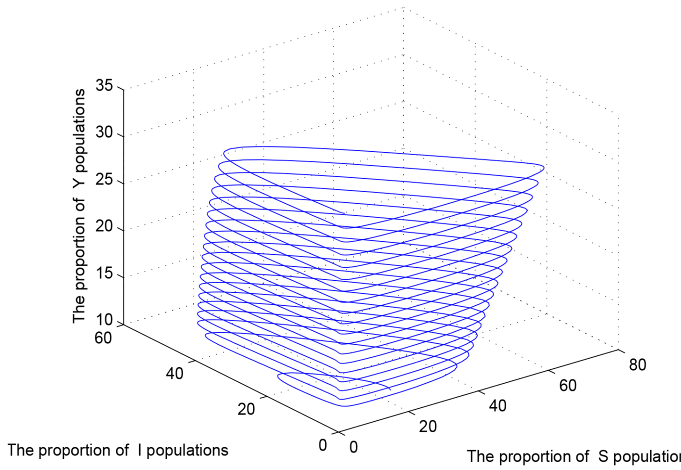


(c)

Figure 1. The time series and the orbits of the system (1.2).



(a)



(b)

Figure 2. The time series and the orbits of the system (1.2).

to unstable stable state see **Figure 2(a)** & **Figure 2(b)**. In other words, the three populations S, I and Y all exist, see **Figure 1(c)**. We can choose different harvesting of either species to control their existence.

Form numerical simulation, the refuge of prey don't influence the system'stability of, fulfilling the conditions of existence and stability of all equilibrium points.

6. Conclusions

This paper discusses an eco-epidemiological model with the disease among the prey, taking into account the prey refuge and a fixed rate of harvesting of every species. All results indicate that harvesting has an important effect on the eco-epidemiological system. Over-exploitation ($E_1 > \frac{r}{q_1}$) would result in the extinction of all populations (susceptible, infected prey and predator). A reasonable harvesting strategy ($\frac{rk-r}{kq_1} < E_1 < \frac{r}{q_1}$) should ensure the existence of the susceptible prey, whereas the infected prey and predator eventually tend to become extinct. And the increasing harvesting E_2 can reduce the quantity of predators and doesn't change the density of infected prey and susceptible prey. However, if $E_2 > \frac{k\beta - c}{q_2}$, the predator dies out, because the increase of E_2 may result in a lack of food for predator. The increased harvesting effort E_3 can result in increasing the quantity of infected prey and a decrease in predator and susceptible prey. The susceptible prey, infected prey, and predator all coexist. If $E_3 > \frac{\mu - d}{q_3}$, the predator becomes extinct. Therefore, it is probable to control the dynamic behavior of the model by choosing reasonable variables E_1, E_2 or E_3.

In the paper, we consider that a fixed rate of harvesting of every species is continuous. In fact, the harvesting of every species is discontinuous. Further studies are required to analyze the dynamics of more realistic but complex systems such as considering the effect of impulsive capturing in different species.

Founding

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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